MATH 131, section 1 – Calculus for Physical and Life Sciences

Review Sheet – Exam 3

November 16, 2007

General Information

- The third exam of the semester will be given on the Wednesday of the week after Thanksgiving break, November 28, from 6:00pm to 7:30pm *in Haberlin 103.* If you have a conflict at that time, we will run the "second seating" at 8:00pm again.
- *Bring a photo ID to the exam.*
- The exam will be designed to take an hour but you will have an extra 30 minutes to work and check your solutions.
- You will be given a TI-30 scientific calculator for the exam which does NOT have graphing capabilities so be prepared to answer questions without your personal calculator. (Note: Some of you may have one of these calculators purchased for use in Chemistry courses here. That is also OK.)
- Use of cell phones, I-pods, and all other electronic devices *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are *turned off*).

What will be covered

The third exam covers sections 3.3, 3.4, 3.5, 3.6, and 3.7 in Chapter 3, plus sections 4.1, 4.2, 4.3, 4.5, and 4.6 in Chapter 4. This includes specifically:

1. Rates of change in the sciences (section 3.3)
2. Derivatives of trigonometric functions (section 3.4)
3. The chain rule for derivatives of compositions (section 3.5)
4. Implicit differentiation (section 3.6), including tangent lines to parametric curves
5. Derivatives of inverse trigonometric and logarithm functions via the chain rule (section 3.7). The method of logarithmic differentiation for functions of the form $u(x)^{v(x)}$ and other forms.
6. Related rates problems (section 4.1)
7. Critical numbers, maximum and minimum values of $f(x)$on $[a, b]$, the first and second derivative tests, applications to curve sketching (sections 4.2 and 4.3)
8. Indeterminate form limits and L'Hopital's Rule (section 4.5)
9. Optimization (max/min) problems (section 4.6)

How to prepare

You should go over the homework problems as well as the text and your class notes. Many of the problems and questions we discuss in class are excellent examples of test questions. I have also listed some sample problems from the text below. Answers can be found in the text.

We will review for the exam in class on Tuesday, November 27. Come prepared with specific questions if there are things you want to discuss.

Some suggested additional review problems

Chapter 3 Review Problems

- 1 - 35 odd numbers. (Note that all the different functions and methods of differentiation that we discussed are included in these, *often combined in various ways.* There will be several similar functions to differentiate on the actual exam and you will need to be able to determine which method to use just by looking at the form of the function.)
- 39, 41, 47 - 57 odds (be sure you understand these)
- 64 (*Answers:* (a) $v(t) = 3t^2 - 12$ and $a(t) = 6t$. (b) upward when $t > 2$, downward when $0 \leq t < 2$. (c) Total distance is 23 (7 up and 16 down). (e) Speeding up for $t > 2$, slowing down for $0 < t < 2$.),
- 65.

Chapter 4 Review Problems

- Graphing, maxima and minima, etc. : 1, 3, 5, 7 (be careful about the domain!), 11, 23
- L'Hopital's Rule: 25, 27, 29, 31
- Related rates: 34 (*Answer:* $\frac{8}{9\pi}$ cm/sec when $h = 5$ cm), 35
- optimization problems: 37, 39, 59

Some Sample Exam Questions

Disclaimer: As always, the actual exam questions may be posed in different ways, may be formatted differently, and may focus on somewhat different aspects of the material we have covered.

I. Find y' and simplify.

(a)

$$y = \ln(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)$$

(b)

$$y = (e^{2x} + 2)^3$$

(c)

$$y = \frac{x+1}{3x^4 - 1}$$

(d)

$$y = \frac{\sin(x)}{1 + \cos(x)}$$

(e)

$$y = \tan^{-1}(e^{5x})$$

(f)

$$xy^2 - 3y^3 + 2x^4 = 2$$

(g)

$$y = \cos(x)^{x^3}$$

II. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$.
(a) Over which intervals with $t \geq 0$ is the amount increasing? decreasing?
(b) Over which intervals is the rate of change of Q increasing? decreasing? decreasing?

III. A spherical balloon is being inflated at 20 cubic inches per minute. When the radius is 6 inches, at what rate is the radius of the balloon increasing? At what rate is the surface area increasing? (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $4\pi r^2$.)

IV. All parts of this question refer to $f(x) = 4x^3 - x^4$.
(a) Find and classify all the critical numbers of f using the First Derivative Test.
(b) Over which intervals is the graph $y = f(x)$ concave up? concave down?
(c) Sketch the graph $y = f(x)$.
(d) Find the absolute maximum and minimum of $f(x)$ on the interval $[1, 4]$.

V. All three parts of this question refer to the function $f(x) = x^{2/3} - \frac{1}{5}x^{5/3}$.
(a) Find all the critical numbers of $f(x)$.
(b) Find all the inflection points of $f(x)$.
(c) For which of the critical numbers here is the Second Derivative Test applicable? Why? Determine the type of each such critical number using the Second Derivative Test.

VI.
(a) A rectangular pen is to be constructed with one side along an existing long stone wall (so no fence is needed along that side), and the other three sides made of new fencing. The pen must enclose 1000 square feet of space. What is the smallest total length of fencing that can be used to enclose the pen? What are the dimensions of the pen that will use the smallest total length of fencing.
(b) Consider all rectangles with one side along the x-axis that are inscribed under the curve $y = \sqrt{x}$, extending to $x = 4$ on the right. Find the dimensions of the rectangle of this type of largest area.

VII. Evaluate the following limits.
(a)

$$\lim_{x \to 0} \frac{1 - \cos(x)}{x^2}$$

(b)

$$\lim_{x \to \infty} \frac{\ln(1 + e^x)}{x}$$