Math 132: Calculus for Physical and Life Sciences 2
Problem Set 9
Due Friday, April 18, 2008, at the beginning of class.

General Directions: You must show all work for credit on these problems.

1. Determine whether each of the following series converges absolutely, converges conditionally, or diverges, using some combination of the Alternating Series Test, the Ratio Test, and the Integral Test.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{e^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(2n+1)}{4n-3}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n7^n}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\ln(n)}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

   (Hint for (e): Try the Ratio Test on this one because of the powers and factorials. The limit you will need to compute is a 1^∞ indeterminate form. These are covered in Section 4.5 of the text if you need to review.)

2. For each of the following power series,

   (i) Determine the radius of convergence R using the Ratio Test.

   (ii) If $0 < R < \infty$, determine whether the series converges at $a \pm R$.

   (iii) Based on (i) and (ii), give the interval of convergence.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

   (c) $\displaystyle\sum_{n=0}^{\infty} n!(x-4)^n$

3. Using the definition, find the Taylor series of each of the following functions $f(x)$ at the indicated a. Write the series using summation notation and also give the first four nonzero terms.

   (a) $f(x) = e^{2x}$, $a = 0$.

   (b) $f(x) = \cos(x)$, $a = \pi/2$.

4. (a) Starting from the geometric series expansion for $f(x) = \dfrac{1}{1+x}$, determine a power series (with $a = 0$) representing the function $\ln(1+x)$.

(b) What is the radius of convergence of your series from part (a)?

(c) Compute the Taylor series of $\ln(1+x)$ at $a=0$. How do your answers to parts (a) and (c) compare?

5. (a) Starting from the Taylor series for $f(t)=\sin(t)$, find a power series representation for the function

$$Si(x)=\int_0^x \frac{\sin(t)}{t}\,dt.$$

(b) Find a power series representation for the function

$$F(x)=\int_0^x \frac{1}{1-t^2}\,dt.$$

6. Compute the requested Taylor polynomials.

(a) The Taylor polynomial of degree $n=3$ for $f(x)=\tan(x)$ at $a=0$.

(b) The Taylor polynomial of degree $n=6$ for $f(x)=\cos(4x)$ at $a=0$.

(c) The Taylor polynomial of degree $n=4$ for $f(x)=\sqrt[3]{x-1}$ at $a=2$.

7. (a) Compute the Taylor polynomial of degree $n=4$ for $f(x)=\sqrt{1+x}$ at $a=0$.

(b) Use your polynomial to approximate $\sqrt{1.1}$. (What x should you use to do this?)

(c) What is the error of your approximation, taking a calculator value for $\sqrt{1.1}$ as the exact value?

8. Use Taylor series, not L'Hopital's Rule, to evaluate each of the following limits.

(a) $\displaystyle\lim_{x\to 0}\frac{1-\cos(x)}{x(e^x-1)}$.

(b) $\displaystyle\lim_{x\to 0}\frac{e^x-e^{-x}-2x}{x-\tan^{-1}(x)}$.

Selected Answers

1.

(a) Converges absolutely.

(d) Converges conditionally (i.e. the series itself converges, but the series of absolute values does not).

(e) Diverges.

2.

(a) (iii) Interval of convergence is $[1, 3)$ (that is all real x with $1 \leq x < 3$).

(c) (iii) Interval of convergence is $\{4\}$ (that is the series converges only for $x = 4$).

3. (b) $\displaystyle\sum_{k=0}^{\infty}(-1)^{k+1}\frac{(x-\pi/2)^{2k+1}}{(2k+1)!}$

4. (a) $\displaystyle\ln(1+x) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}x^n}{n}$ for all x with $|x| < 1$.

5. (b) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}$.

6. (b) $\displaystyle p_6(x) = 1 - 8x^2 + \frac{32x^4}{3} - \frac{256x^6}{45}$.

8. (b) 1.