Math 132: Calculus for Physical and Life Sciences 2
Problem Set 8
Due Friday, April 11, 2008, at the beginning of class.

General Directions: You must show all work for credit on these problems.

1. Determine whether each of the following sequences converges or diverges. If it converges, find its limit.

   (a) $a_n = \dfrac{1-n^2}{2+3n^2}$

   (b) $a_n = 1 + (-1)^n$

   (c) $a_n = \dfrac{\sin n}{3^n}$

   (d) $a_n = \dfrac{(\ln n)^2}{n}$

   (e) $a_n = \dfrac{2^n+1}{e^n}$

   (f) $a_n = \ln(2n+1) - \ln(2n-1)$

2. Determine if each infinite series converges or diverges. If it converges, find its sum. (Be careful about the first term of the series.)

   (a) $1 + e^{-1} + e^{-2} + e^{-3} + \ldots + e^{-n} + \ldots$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{3^n - 2^n}{4^n}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{100}{99}\right)^n$

   (d) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{99}{100}\right)^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{e}\right)^n$

   (f) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{1}{2^n}\right)$

   (g) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k+1}{2k+4}\right)$

   (h) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$

   (Hint for part (h): Consider several partial sums and use properties of logarithms to simplify them.)

3. Find the rational number represented by $0.2525252525\ldots$.

   (Hint: Think of $0.2525252525\ldots$ as a geometric series.)

4. Suppose the government spends \$1 billion and that each recipient spends 90% of the dollars that he or she receives. In turn, the secondary recipients spend 90% of the dollars they receive, and so on. How much total spending results from the original injection of \$1 billion into the country?

5. Use the integral test to decide whether each of the following series converges or diverges.

(a) $\sum_{n=0}^{\infty} \frac{n}{n^2+1}$ (b) $\sum_{n=0}^{\infty} \frac{n}{e^{n^2}}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ (d) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

(e) $\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right)$

(Hint for part (e): integrate by parts.)

6. Determine whether the given series converges or diverges. Quote general results to justify your answers.

(a) $\sum_{n=1}^{\infty} \frac{1}{(\sqrt{n})^5}$ (b) $\sum_{n=1}^{\infty} \frac{n+1}{n^2}$

7. Show that the series $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

8. For each of the following series explain why the integral test does *not* apply.

(a) $\sum_{n=1}^{\infty} e^{-n} \sin n$ (b) $\sum_{n=1}^{\infty} \frac{2+\sin n}{n^2}$

Selected answers:

1. (a) sequence converges to $-\frac{1}{3}$, (c) sequence converges to 0, (f) sequence converges to 0.
2. (b) series converges and the sum is 2, (f) series diverges, (h) series diverges.
3. $\frac{25}{99}$

5. (b) series converges, (d) series converges.
6. (a) series converges.