Math 132: Calculus for Physical and Life Sciences 2
Problem Set 7
Due Friday, April 4, 2008, at the beginning of class.

General Directions: You must show all work for credit on these problems.

1. All parts of this question deal with the differential equation $y' = x + y$.
   (a) Verify that $y = -x - 1$ is a solution of this differential equation.
   (b) Sketch the direction field of this equation by hand, showing the slopes at all points with $-3 \le x \le 3, -3 \le y \le -3$ with integer coordinates.
   (c) On your direction field graph, sketch approximate graphs of the solutions of this differential equation satisfying the initial conditions $y(0) = -2$, $y(0) = 0$, $y(0) = 1$.
   (d) Use Euler's method with $n = 5$, $\Delta x = .2$ to approximate the solution of the equation with the initial condition $y(0) = 1$ for $0 \le x \le 1$.

2. All parts of this question deal with the differential equation $y' = 3 - y$.
   (a) Sketch the direction field of this equation by hand, showing the slopes at all points with $-1 \le x \le 5, 0 \le y \le 5$ with integer coordinates.
   (b) On your direction field graph, sketch approximate graphs of the solutions of this differential equation satisfying the initial conditions $y(0) = 0$, $y(0) = 3$, and $y(0) = 4$. What is the long term behavior of each?
   (c) Use Euler's method with $n = 5$, $\Delta x = .2$ to approximate the solution of the equation with the initial condition $y(0) = 1$ for $0 \le x \le 1$.
   (d) This is a separable equation. Derive the general solution by separating variables and integrating.
   (e) What is the exact formula for the solution of the differential equation with the initial condition $y(0) = 1$?

3. Solve each of the following differential equations, finding an explicit formula for y as a function of x if possible (you can leave the solution in implicit form if not). In some cases you are given initial conditions; use them to determine the arbitrary constant in the general solution.
   (a) $y' = \dfrac{\tan(y)}{x^2 + 4x + 3}$, find the general solution.
   (b) $y' = \dfrac{(x-1)y^5}{x^2(2y^3 - y)}$, find the general solution.
   (c) $y' = 2xy$ with $y(0) = 4$.
   (d) $y' = x(y^2 + 1)$ with $y(0) = 1$.
   (e) $y' = 2xy^2 + 3x^2y^2$ with $y(1) = -1$.

(f) $y' = 3y(1 - y/20)$ with $y(0) = 5$. (Note: this is a logistic equation.)

4. (Radiocarbon dating.) C^{14} is a radioactive isotope of carbon with half-life of approximately 5700 years. It occurs naturally and is incorporated in living tissues by normal metabolic processes. Assume that any C^{14} present when the tissue dies decays exponentially from that time on. Carbon extracted from an ancient skull contains only one-sixth as much C^{14} as carbon extracted from present-day bone. How old is the skull?

5. (Newton's Law of Heating/Cooling.) A pitcher of buttermilk initially at 25° C is cooled by setting it out on the front porch on a cold day when the exterior temperature is a constant 0° C. Suppose that the buttermilk has reached 15° C after 20 minutes.

   (a) What will the temperature be after 25 minutes?

   (b) When will the temperature reach 5° C?

6. The population of fish in a lake is attacked by a microscopic water-borne parasite at $t = 0$, and as a result the population declines at a rate proportional to the *square root* of the population from that time on.

   (a) Express this statement about the rate of growth of the population P as a differential equation. (There should be a constant of proportionality, say $-k$, in your equation.)

   (b) Use separation of variables to find the general solution of your differential equation.

   (c) At $t = 0$ there were 900 fish in the lake; 441 were left after 6 weeks. When did the fish population disappear entirely?