Math 132: Calculus for Physical and Life Sciences 2
Problem Set 5
Due Friday, February 29, 2008, at the beginning of class.

General Directions: You must show all work for credit on these problems.

1. Explain why each of the following integrals is an improper integral and determine whether each one converges or diverges. Evaluate those that are convergent.

   (a) $\displaystyle\int_4^{\infty} \frac{1}{x^{3/2}}\,dx$ (b) $\displaystyle\int_0^4 \frac{1}{\sqrt{4-x}}\,dx$

   (c) $\displaystyle\int_0^2 \frac{x}{x^2-1}\,dx$ (d) $\displaystyle\int_e^{\infty} \frac{1}{x\ln x}\,dx$

   (e) $\displaystyle\int_0^{\infty} xe^{-x^2}\,dx$ (f) $\displaystyle\int_0^{\infty} \sin^2 x\,dx$

   (g) $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ (e) $\displaystyle\int_{1/2}^1 \frac{2}{\sqrt{1-x^2}}\,dx$

2. (i) Sketch the given curves and determine where they intersect.

   (ii) Find the area bounded by the given curves.

   (a) $y=-x^2$ and $y=x^2-2x$

   (b) $y=3x^2+3x$, $y=2x+4$, $x=-1$, and $x=3$

   (c) $y=\dfrac{5}{x}+x$ and $y=6$ (in the region $x>0$)

   (d) $y=\sqrt{x}$ and $y=x-2$, $(x>0)$

   (e) $y=e^x$, $y=e^{-x}$, $x=0$, and $x=1$

   (f) $x=2-y^2$ and $x=y^2$

3. Consider the curve $y=\dfrac{1}{x^2}$ for $1\le x\le 6$.

   (a) Calculate the area under this curve on the given interval.

   (b) Determine c so that the line $x=c$ bisects the area of part (a). (*I.e.*, it divides the region into two regions of equal area.)

   (c) Determine d so that the line $y=d$ bisects the area of part (a).

Selected answers:

1. (a) Infinite interval of integration. Integral converges to 1.
   (c) Infinite discontinuity (vertical asymptote) at $x = 1$. Integral is divergent.
   (e) Infinite interval of integration. Integral converges to $1/2$.
   (g) Infinite discontinuity (vertical asymptote) at $x = 0$. Integral converges to $2e - 2$.
2. (a) $1/3$
   (c) $12 - 5\ln 5$
   (e) $e + 1/e - 2$
3. (c) $d = \dfrac{17}{12} - \dfrac{1}{3}\sqrt{15}$