

Math 132: Calculus for Physical and Life Sciences 2
Extra Credit Problem Set 10
Due Wednesday, April 30, no later than 5:00pm

General Directions: This is an optional problem set. You may submit solutions for some or all of the following problems. Any points you earn will count as extra credit on the problem set component of your semester average. You must show all work for credit on these problems.

1. The metal making up a rod has density $\rho(x) = 4 + \sin(x)$ at location x . The rod extends from $x = 0$ to $x = \frac{3\pi}{2}$ along the x axis. Determine the total mass and the location of the center of mass of the rod.
2. A metal plate with constant density 5 gm/cm^2 has the shape of the region bounded by $y = \sqrt{x^2 + 1}$ and the x -axis, for $0 \leq x \leq 4$. Find the location of the center of mass of the plate.
3. The probability of a certain type of transistor failing between $t = a$ and $t = b$ (months), for $0 < a < b$, is given by

$$P(a \leq t \leq b) = c \int_a^b e^{-ct} dt.$$

- (a) If the probability of failure within the first six months is .10, what is the value of c ?
 - (b) Given the value of c in part (a), what is the probability that the transistor lasts at least 6 months?
 - (c) Given the value of c in part (a), what is the *mean* life of this type of transistor?
 - (d) Given the value of c in part (a), what is the *median* life of this type of transistor?
4. Suppose that t is the time (in hours) it takes for calculus students to complete their final exam. Assume that all students finish within 3 hours and that the probability density function for the time t is

$$f(t) = \begin{cases} \frac{4x^3}{81} & \text{if } 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

- (a) What proportion of the students take between 1.5 and 2.5 hours to complete the exam?
- (b) What is the *mean time* for students to complete the exam?
- (c) What is the *median time* for students to complete the exam?
- (d) At what time should the (evil) professor set the end of the exam period if he wants to make sure that only $2/3$ of the students have completed the exam when the papers are collected?

5. The distribution of scores on IQ exams is often modeled by a *normal* distribution with mean $\mu = 100$ and standard deviation $\sigma = 15$.
- (a) Give the formula for the normal pdf that fits this description.
 - (a) Estimate the fraction of the population with IQ scores between 115 and 120 by applying a Midpoint Riemann sum approximation for the appropriate integral. Use $n = 5$ subintervals in the Riemann sum.
 - (b) Estimate the fraction of the population with IQ scores between 140 and 150 by the same method as in part (b).
6. Let μ and $\sigma^2 > 0$ be any two real constants.
- (a) Show that the normal pdf

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- has exactly one critical number at $x = \mu$, and that $f(x)$ has a local maximum at $x = \mu$.
- (b) Show that $f(x)$ has inflection points at $x = \mu \pm \sigma$.
 - (c) Give qualitative sketches of $y = f(x)$ with $\mu = 4$ and $\sigma^2 = 4$, and the cumulative distribution function for this normal distribution.