College of the Holy Cross, Fall Semester, 2004
Math 131
Worksheet for Thursday, December 2

1. Write out (but don't calculate) the left hand sum and right hand sum for the function $f(x) = e^{-x}$ over the interval $-1 \leq x \leq 1$ using a partition consisting of $n = 4$ subintervals of equal length. Which of these is an underestimate to the exact area under the graph? Which is an overestimate? Which is a better estimate? (Look at the concavity of the graph of f.)

2. Express the integral
$$\int_0^{2\pi} \sin(x)\, dx$$
as a sum/difference of areas. Draw a picture and label the relevant areas. What is the exact value of the integral?

3. Write down an integral that represents the area of the region that is bounded below by the x-axis and above by the curve $y = 3x - x^2$. You'll need to sketch the curve first and find its roots.

4. Suppose that a population of birds on an island grows at a rate of
$$\frac{4500e^{-0.05t}}{(1 + 9e^{-0.05t})^2} \quad \text{birds/year,}$$
where t is measured in years since 2000. If the population in the year 2000 was 1000 birds, what is the expected population in the year 2015? Express the population as an integral, but don't attempt to calculate it.

5. What integral represents the average value of the function $f(x) = \sin(x)$ over the interval $0 \leq x \leq \pi$. Show that the average is at most 1 and at least $1/2$. Hint: The region under the graph of $\sin x$ is contained within a rectangle, and contains a triangle.

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