College of the Holy Cross, Spring Semester, 2005
Math 132 Problem Set Style Guide

A well-written problem set, lab report, or test paper can be read and understood by someone familiar with the material but not necessarily enrolled in the course. You should strive to provide context in the work you submit for grading. For example, it's helpful to summarize the question you're answering (on a problem set or lab report), and essential to write a "narrative", not just to scrawl a sequence of equations.

The test solutions distributed last semester suggest the style we hope you'll achieve. A couple of sample book questions are answered below in the same spirit. You needn't type your problem sets, of course, but if you do, please see your instructor for software recommendations. (Word processors are a poor choice.)

Section 4.6, #13 Find the coordinates of the point on the parabola $y = x^2$ that is closest to the point $(3, 0)$.

Solution The distance from (x, y) to $(3, 0)$ is $\ell = \sqrt{(x-3)^2 + y^2}$. Since $y = x^2$ on the parabola, we may express the distance as

$$\ell = \sqrt{(x-3)^2 + x^4}.$$

The distance is smallest exactly when the square of the distance is smallest, so we may as well minimize

$$f(x) = \ell^2 = x^4 + x^2 - 6x + 9.$$

To find the critical point(s), we set the derivative equal to zero: $f'(x) = 4x^3 + 2x - 6 = 0$. By inspection, $x = 1$ is a critical point, and factoring gives $f(x) = (x-1)(4x^2 + 4x + 6)$. The discriminant of the quadratic factor is $b^2 - 4ac = 4^2 - 4 \cdot 4 \cdot 6 < 0$, so there are no other critical points of f. The closest point to $(3, 0)$ on the parabola $y = x^2$ is therefore $(1, 1)$.

Section 5.4, #11 (a) Let $\int_0^3 f(x)\,dx = 6$. What is the average value of f on $[0, 3]$?

(b) If f is even, what is $\int_{-3}^3 f(x)\,dx$? What is the average value of f on $[-3, 3]$?

Solution (a) The average value of f on $[a, b]$ is $\frac{1}{b-a}\int_a^b f(x)\,dx$. In this example, the average value is $\frac{1}{3-0}\int_0^3 f(x)\,dx = 6/3 = 2$.

(b) If f is even, then $\int_{-3}^3 f(x)\,dx = 2\int_0^3 f(x)\,dx = 12$ by a formula on page 246 of the text. Similarly to part (a), the average value is $\frac{1}{3-(-3)}\int_{-3}^3 f(x)\,dx = 12/6 = 2$.