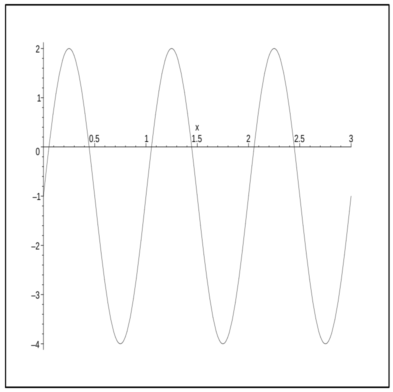
Mathematics 131 – Calculus for Physical and Life Sciences 1
Practice Final Examination
December 10, 2004

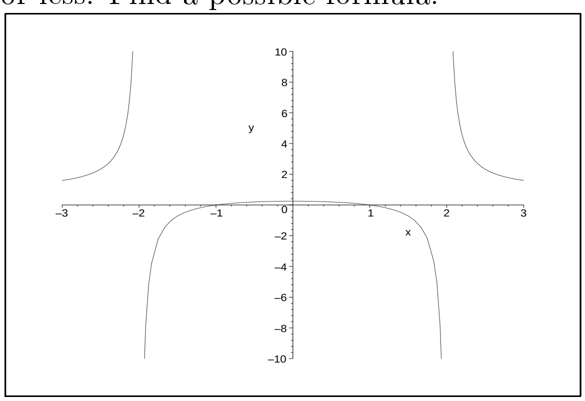
I.

- A) Give a possible formula for the function plotted here:



- B) Does the function from part A have an inverse function if x is restricted to the interval $0 \leq x \leq 1$? If so, say why and give the domain of the inverse function. If not, say why not.

II. The function plotted here is a rational function with numerator and denominator both polynomials of degree 2 or less. Find a possible formula.



III. One of the functions tabulated below is approximately linear and the other is approximately exponential. Say which is which and give a formula for either one (your choice).

x	.2	.4	.6	.8	1.0
$f(x)$	2.30	2.51	2.72	2.93	3.14
$g(x)$	2.30	2.42	2.55	2.69	2.84

IV. A cup of hot chocolate is set out on a counter at $t = 0$. The temperature of the chocolate t minutes later is $70 + 80e^{-t/3}$ (in degrees F).

- A) What is the temperature of the chocolate at $t = 0$?
- B) How fast is the temperature changing at $t = 10$ (give units).
- C) How long does it take for the temperature to reach 100°F?

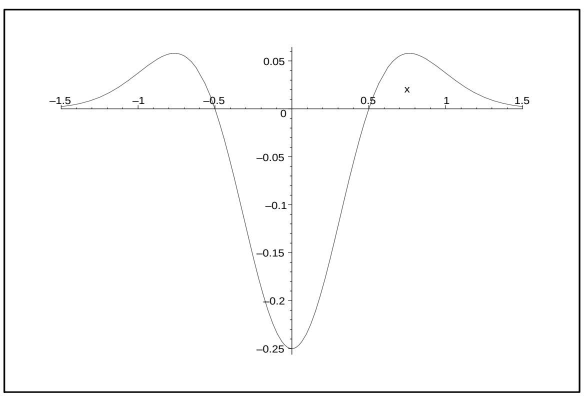
V.

- A) Using the limit definition, compute $f'(x)$ for $f(x) = \frac{1}{x+1}$. Using appropriate derivative rules, compute the derivatives of the following functions:
- B) $g(x) = 3x^4 + \frac{3}{\sqrt{x}} + 2\sqrt[3]{x} + \pi^2$.
- C) $g(x) = \frac{\tan(x)+x}{\cos(2x)}$
- D) $i(x) = 3\ln(x^2 + 3^x)$
- E) $j(x) = \arcsin(12x + 2)$

VI.

- A) Give parametric equations for the circle of radius 2 centered at $(0, 0)$, traversed counterclockwise with constant speed 1.
- B) An object moves along the path given by your parametric curve in part A. Give a function of t that computes the object's distance from the point $(1, 3)$ at each time t.

VII. The following graph shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $-3/2 \leq x \leq 3/2$.



Using the graph, *estimate*

- A) The intervals on which f is increasing.

- B) The critical points of $f(x)$. Say what the type (local max, local min, neither) of each critical point is.
- C) The intervals on which $y = f(x)$ is concave up.
- D) The locations of the inflection points of $y = f(x)$.

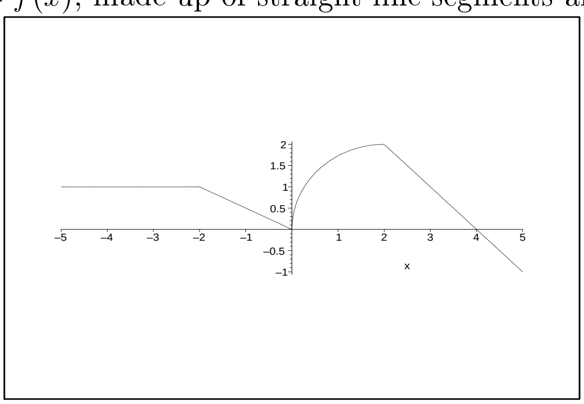
VIII. *Note: The actual exam will have only one of this type of problem!*

- A) A window is to be made of a single large piece of glass in the shape of a semicircle on top of a rectangle (the diameter of the semicircle is the same as the width of the rectangle). The perimeter of the window is 30 ft. What dimensions will maximize the total area of the window (and the light admitted)?
- B) A town wants to build a water pumping station on a small island 2 miles from the straight shoreline of the lake that forms its water reservoir. The town is 6 miles along the shore from the point nearest to the island. The town must be connected to the pumping station by a pipeline made up of straight line segments. It costs \$3 million per mile to lay pipe under the water and \$2 million per mile to lay pipe along the shoreline. Where along the shoreline should the pipeline hit land to minimize the costs of construction?

IX. *Note: The actual exam will have only one of this type of problem!*

- A) A rocket rises vertically from a point on the ground that is 100 m from an observer at ground level. When the angle of elevation of the rocket is $2\pi/3$, the angle of elevation is increasing at a rate of $2\pi/15$ radians per second. How fast is the rocket moving at that time?
- B) An oil spill from a tanker accident has happened in Pristine Bay. The slick has the form of a (short) circular cylinder floating on the water surface. Oil-eating bacteria are being used to contain the slick, and they are gobbling 5ft^3 of oil per hour. When the radius of the slick is 500 ft, the thickness is .01 ft and the thickness is decreasing at a rate of .001 ft/hour. How is the radius of the slick changing at that time?

X. This is the graph $y = f(x)$, made up of straight line segments and a semicircle:



- A) Use left- and right-hand sums with $\Delta x = 2$ to estimate $\int_{-5}^{5} f(x)\ dx$.
- B) Using the relation between the integral and area, find the *exact* value of $\int_{-5}^{5} f(x)\ dx$.