Solutions

1. Solve for $x$: $3x^2 - 21x + 30 = 0$.

Method 1 (easier): Factoring, we see $3x^2 - 21x + 30 = 3(x - 2)(x - 5)$. This equals zero only when one of the factors is zero. $3 \neq 0$, so the roots are given by

$$x - 2 = 0 \Rightarrow x = 2,$$
$$x - 5 = 0 \Rightarrow x = 5.$$

Method 2 (more calculation): Use the quadratic formula: $ax^2 + bx + c = 0$ has roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Here $a = 3$, $b = -21$, $c = 30$, so

$$x = \frac{21 \pm \sqrt{441 - 4(3)(30)}}{6} = \frac{21 \pm \sqrt{81}}{6} = \frac{21 + 9}{6}, \frac{21 - 9}{6} = 5, 2.$$

Note: You could also factor out the 3, then apply the quadratic formula.

2. Let $f(x) = \sqrt{x - 1}$ and $g(x) = x^3 + 1$. Find $f(g(x))$? What is the domain of $f(g(x))$?

$$f(g(x)) = \sqrt{g(x) - 1} = \sqrt{(x^3 + 1) - 1} = \sqrt{x^3} = x^{3/2}$$

The domain of a function defined this way is taken to be the biggest set of real numbers you can substitute into the formula and get a well-defined result. Here, we can’t take the square root of a negative number and get a real value, so $x^3 \geq 0$ is necessary. This is the same as $x \geq 0$. The domain is the set of all $x \geq 0$ in the real numbers.

3. Simplify and express as a single fraction:

$$\frac{x}{\sqrt{x+1}} + \sqrt{x+1}$$

First we rewrite the fraction as multiplying by the reciprocal of the denominator, then put the terms in the numerator over a common denominator so they can be added, then finally
combine:
\[
\frac{x}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{x^2+3} = \left( \frac{x}{\sqrt{x+1}} + \frac{\sqrt{x+1}}{x} \right) \cdot \left( \frac{x^2+3}{x} \right)
\]
\[
= \left( \frac{x}{\sqrt{x+1}} + \frac{x+1}{\sqrt{x+1}} \right) \cdot \left( \frac{x^2+3}{x} \right)
\]
\[
= \left( \frac{2x+1}{\sqrt{x+1}} \right) \cdot \left( \frac{x^2+3}{x} \right)
\]
\[
= \frac{(2x+1)(x^2+3)}{x\sqrt{x+1}}
\]

(Many other forms possible too.)

4. Let \( f(x) = x^2 + 2x \). Compute and simplify: \( f(2 + h) - f(2 - h) \).

In an expression like \( f(2 + h) \), we take the “inside” (the \( 2 + h \)) and substitute that in for \( x \) everywhere in the definition of the function:
\[
f(2 + h) - f(2 - h) = [(2 + h)^2 + 2(2 + h)] - [(2 - h)^2 + 2(2 - h)]
\]

Now expand, collect like powers and simplify:
\[
[(2 + h)^2 + 2(2 + h)] - [(2 - h)^2 + 2(2 - h)] = [4 + 4h + h^2 + 4 + 2h] - [4 - 4h + h^2 + 4 - 2h]
\]
\[
= [8 + 6h + h^2] - [8 - 6h + h^2]
\]
\[
= 12h
\]