Mathematics 131 – Calculus for Physical and Life Sciences
Lab Day 4 – Families of Curves
November 12, 2004

Background

We have now introduced a number of tools relating properties of the derivatives of a function f to the properties of the function f itself (the First and Second Derivative Tests for critical points, and so forth). These are also useful in situations where we want to study not just one curve, but a whole *family* of curves, defined by a formula containing one or more "parameters" (constants that can take on different values). In this lab we will study several examples of these families of curves, and see how the parameters can influence the shape of the curve.

Lab Questions

A) First we will study the family of curves defined by

$$y = f(x) = e^{-(x-a)^2/b}.$$

Here a and $b > 0$ are the two parameters in the formula. To work with all of these simultaneously, we can define a function in Maple like this:

```
f := (x,a,b) -> exp(-(x-a)^2/b);
```

To plot individual curves in the family, we use the plot command as before, and supply the particular a and b-values for the curve in the family that we want. For instance, to plot the curve with $a = 0$, $b = 1$ on the interval $-3 \leq x \leq 3$, for instance, we would use

```
plot(f(x,0,1),x=-3..3);
```

1) Plot the curves in the family for $a = 0$ and $b = .1, 1, 3, 10$ together on the same coordinate axes. Pick an appropriate range of x-values to plot so that you are seeing all of the graphs in reasonable detail. (The lab sheet for Lab Day 1 includes information about how to do this in Maple if you don't remember. Recall that you can also specify which color to use for each plot – this will help to tell which is which.) Describe what happens if we vary b but keep a fixed in this family.
2) Now plot the curves in the family for $a = -2, -1, 0, 1, 2$ together on the same coordinate axes. Again, pick an appropriate range of x-values to plot so that you are seeing all of the graphs in reasonable detail. Describe in general what happens if we vary a but keep b fixed.
3) How does the location of the critical point(s) of $f(x)$ depend on a, b in this family? In Maple, you can compute $f'(x)$ by using `diff(f(x,a,b),x)`. Where is this equal to zero, or undefined? Does this match your plots in parts 1,2? Explain.
4) Where do the curves in this family have inflection points, and how does their location depend on a, b?

B) Now consider the family of curves defined by

$$y = g(x) = x^3 - 3ax + 1.$$

(Just one parameter, a, here.)

1) Plot the curves in the family for $a = -2, -1, 0, 1, 2$ on the same coordinate axes.
2) How do the number and type of the critical points in this family depend on the value of a?
3) There is exactly one value of a for which $g(x)$ has a *double root*. Find this a and plot the corresponding curve in the family to check your work.

C) Finally, we consider the family of curves given by

$$y = h(x) = \frac{1}{(x-a)^2 + b}$$

(two parameters, a and b here).

1) Fix $a = 0$ and the curves in this family for $b = -2, -1, -0.2, 0, 0.2, 1, 2$. Plot all five curves together on the same axes. You will note that some of these have vertical asymptotes. To "cut off" the graphs and get reasonable pictures, add the options `y=-20..20,discont=true` in the plot command.
2) Now fix b (you get to choose the value) and vary a in this family. Plot four curves together on the same axes.
3) For what values of a and b does the graph $y = h(x)$ have vertical asymptotes? Where are the asymptotes located (give in terms of a, b).
4) Where are the critical point(s) of $y = h(x)$ located. If you fix a, does the type of critical point (maximum, minimum, neither) depend on the value of b? Explain.
5) Find values of a, b so that the function $y = h(x)$ has a local maximum at the point $(x, y) = (3, 5)$. Plot your curve.
6) Where do the curves in this family have inflection points, and how does their location depend on a, b?

Assignment

One write-up from each lab group. Due: Wednesday, November 17.