Mathematics 131, section 1 – Calculus for Physical and Life Sciences
Discussion 1 – "New Functions From Old"
September 6, 2004

Working in a Group

Since this is the first of the discussion class of the semester, a few words about this way of working are probably in order. In the discussion meetings of this class, we will be aiming for *collaborative learning* – that is, for an integrated group effort in analyzing and attacking the discussion questions. The ideal is for everyone in each of the groups to be fully involved in the process. By actively participating in the class through talking about the ideas yourself in your own words, you can come to a better first understanding of what is going on than if you simply listen to someone else (even me!) talk about it.

However, to get the most out of this kind of work, some of you may have to adjust some of your preconceptions. In particular:

- This is *not a competition*. You and your fellow group members are working as a team, and the goal is to have everyone understand what the group does fully.
- At different times, it is inevitable that different people within the group will have a more complete grasp of what you are working on and others will have a less complete grasp. Dealing with this a group setting is excellent preparation for real work in a team; it also offers opportunities for significant educational experiences.
- If you feel totally "clueless" at some point and everyone else seems to be "getting it," your job will be to ask questions and even pester your fellow group members until the point has been explained to your full satisfaction. (Don't forget, the others may be jumping to unwarranted conclusions, and your questions may save the group from pursuing an erroneous train of thought!)
- On the other hand, when you think you do see something, you need to be willing to explain it patiently to others. (Don't forget, the absolutely best way to make sure you really understand something is to try to explain it to someone else. If you are skipping over an important point in your thinking, it can become very apparent when you set out to convey your ideas to a team member.)

In short, *everyone has something to contribute, and everyone will contribute in different ways at different times.*

Mathematical Goals

Once we have some function f, we can create a number of new functions from it by simple transformations:

- we can multiply f by a constant c to produce a new function g with $g(x) = cf(x)$,
- we can add a constant c to f to produce a new function g with $g(x) = f(x) + c$,
- we can substitute $x + c$ into the function f to produce a new function $g(x) = f(x + c)$,
- we can substitute cx into the function f to produce a new function $g(x) = f(cx)$.

Today we want to recall (or perhaps figure out for the first time!) what these transformations do *in geometric terms*. In other words, we want to understand the effect of each on the graph $y = f(x)$. The key question is, given the graph $y = f(x)$, how is the graph $y = g(x)$ obtained in each of the four cases above?

Discussion Questions

I. The graph below shows a part of the graph $y = f(x)$ for a function with domain equal to the whole set of real numbers.

A) Sketch the graphs $y = f(x) + 1$ and $y = f(x) - 2$, showing all x- and y-axis intercepts and asymptotes.
B) Sketch the graphs $y = 2f(x)$ and $y = \frac{-2}{3} f(x)$, showing all x- and y-axis intercepts and asymptotes.
C) Sketch the graphs $y = f(x+1)$ and $y = f(x-1)$, showing all x- and y-axis intercepts and asymptotes.
D) Sketch the graphs $y = f(2x)$ and $y = f(-x/4)$, showing all x- and y-axis intercepts and asymptotes.

II. Now in your own words, in complete English sentences, explain the general geometric relation between each of the following. Note: You should be looking for things that are true for all functions. The examples in question I should be suggestive, but don't focus only on properties of that $f(x)$.
A) The graphs $y = f(x)$ and $y = f(x) + c$: How is $y = f(x) + c$ obtained from $y = f(x)$, and what difference does the sign of c (i.e. whether $c > 0$ or $c < 0$) make?
B) The graphs $y = f(x)$ and $y = cf(x)$: How is $y = cf(x)$ obtained from $y = f(x)$, and what difference do the sign and magnitude of c make? (That is, how is the case $c < 0$ different from $c > 0$? How is the case $|c| < 1$ different from $|c| > 1$?)
C) The graphs $y = f(x)$ and $y = f(x + c)$: How is $y = f(x + c)$ obtained from $y = f(x)$, and what difference does the sign of c make?
D) The graphs $y = f(x)$ and $y = f(cx)$: How is $y = f(cx)$ obtained from $y = f(x)$, and what difference do the sign and magnitude of c make? (That is, how is the case $c < 0$ different from $c > 0$? How is the case $|c| < 1$ different from $|c| > 1$?)

Assignment

Group writeups due in class Wednesday, September 8.