Mathematics 132 – Calculus for Physical and Life Sciences 2
Lab Day 2 – Applications of Integration
March 15, 2005

Background

Recall that in class yesterday, we derived an integral formula for the arc length L of the portion of the graph $y = f(x)$ from $x = a$ to $x = b$:

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx.$$

Today, we want to apply this to a "real-world" problem.

Maple Background

The integrals you need to compute today are ones that can be expressed in elementary form using the functions we know. If you want a symbolic formula for an indefinite integral, you can either use our hand computation methods, the table of integrals, or Maple's `int` command. For instance,

```
int(sqrt(1+x^2),x);
```

asks Maple to find $\int \sqrt{1+x^2}\, dx$ in symbolic form.

As in Lab Day 1, we will also want to compute numerical approximations to integrals for today's lab. Instead of comparing how accurate different integration methods are, though, we simply want to let Maple determine an accurate value. So for any integrals to be evaluated today, just use the method you used for the "exact" values in Lab Day 1. For instance, if you wanted to compute $\int_0^2 e^{\sin(x)}\, dx$, you would use:

```
evalf(Int(exp(sin(x)),x=0..2));
```

If you want to graph a function like $y = e^{\sin(x)}$, use the basic `plot` command:

```
plot(exp(sin(x)),x=0..2);
```

Maple also has numerical routines for finding approximate solutions of various types of equations. The basic command here is called `fsolve`. For instance, suppose we wanted to solve the equation $x - e^{\sin(x)} = 4$. We could do this as follows. First we set up the equation to be solved:

```
eq:=x-5*sin(x) = 4;
```

Then the `fsolve` command will determine a solution:

```
fsolve(eq,x);
```

If you have reason to believe that your equation has more than one solution (the equation $x - 5\sin(x) = 4$ has three!) and you want to look for one *near a given x-value*, for instance near $x = 4$, use a command like this:

```
fsolve(eq,x=4);
```

Lab Problems

The distance between the towers of the main span of the Golden Gate Bridge in San Francisco is about 1280 m. The "sag" of the cables supporting the roadway (the vertical distance from the top of the towers down to the lowest point of the cable halfway between the towers) is about 143 m. on a cold winter day.

A) If we ignore the fact that the roadway is pulling the cables downward (in addition to the weight of the cables themselves), then the shape of the cables would be that of a curve called a *catenary*. The general equation of a catenary is

$$y = c(x) = \frac{e^{ax} + e^{-ax}}{2} \tag{1}$$

for some constant a.

1) First, let's determine a catenary that matches the dimensions of the Golden Gate Bridge. We will choose coordinates so the lowest point on the cable is at $y = 0$. The general equation (1) always gives $c(0) = 1$, so we need to shift down and consider:

$$y = c(x) = \frac{e^{ax} + e^{-ax}}{2} - 1$$

To determine the correct value for a, we want to solve the equation $c(640) = 143$. We can do this in Maple as follows:

```
cat:=x->(exp(a*x)+exp(-a*x))/2 - 1;
eq:=cat(640) = 143;
avalue:=fsolve(eq,a);
```

Note: `cat` is a function whose graph is the shifted catenary. The output from the last command will be the appropriate value of a.

2) Plot the catenary with the value of a you found in part 1, and verify that you have a plausible curve for the shape of a hanging cable. You can substitute the value for a into the expression using a command:

```
cats:=subs(a=avalue,cat(x));
```

3) Determine the arclength of the catenary cable, using the following:

```
length:=evalf(Int(sqrt(1+diff(cats,x)^2),x=-640..640));
```

(The first line substitutes the value for a and calls the resulting expression `cats`. The second computes the arclength integral for the function using

$$L = \int_{-640}^{640} \sqrt{1 + \left(\frac{dcats}{dx}\right)^2}\, dx$$

B) The shape of the "loaded" cables of a suspension bridge is actually approximately *parabolic*. Find the equation of an appropriate *parabola* that approximates the shape of the cables of the Golden Gate Bridge. Note: You'll need to decide how to introduce coordinates in the diagram above. The origin of the xy-plane can be anywhere in the picture that is convenient. Explain your choice in a text region. When you have your equation, plot your function using Maple to check that its graph matches the diagram above(!)

C) Using your parabola's equation, determine the *length of the cables* (that is the length from one tower to the other along the parabola with the shape of the hanging cable). Note: You'll need to set up the function to integrate. That can be done "by hand" or by using some of Maple's built-in commands. Look at the Help page for the `diff` command, for instance.

D) As a check on your answer in part C, also compute the length of the straight line joining the top of the left tower to the lowest point of the cable, then add the length of the line back up to the top of the right tower. Is your answer in B consistent with this? Explain.

E) On a hot summer day, because of thermal expansion of the metal, the cable is about 0.05% longer (but the shape of the cable is still parabolic). The effect of the heat on the concrete of the towers is negligible. *How much does the "sag" of the cable increase?* Would you be likely to notice this if you drove over the bridge?

Assignment

Writeups due in class on Monday, March 21.