Mathematics 132 – Calculus for Physical and Life Sciences 2
Things to Know – Final Exam
May 2, 2005

General Information

- The final exam will be given *in our regular class room, Swords 328* from 2:30pm to 5:30pm on Tuesday, May 10.
- This will be a *comprehensive exam* – it will cover all of the material from Chapters 5-11 of the text that we have studied this semester. See the detailed list of topics below.
- The final will be similar in format to the three midterm exams, but roughly *1.75 times as long.*
- If you are well prepared and work steadily, it should be possible to complete the exam in about 2 to 2.5 hours. But you will have the full 3 hour period to work on the exam if you need that much time.
- If there is interest, we can up a daytime or evening review session. *Important Note:* I will be out of town from about 2:00pm on Wednesday, May 4 through the end of the day on Friday May 6. I'll be visiting Oberlin College as their Mathematics External Honors Examiner, so even though I may be able to check email, I probably will not have much (any!) free time. I will be returning to the East Coast on Friday evening, though, so we could do a review session any time Sunday or possibly Monday evening. We will discuss this in class on Monday, May 2.

Topics to Be Covered

1) The definite integral, connections with areas, average values, total change.
2) The Fundamental Theorem of Calculus: If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $[a, b]$, then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

3) Antiderivatives graphically and numerically
4) The power, sum, and constant multiple rules for antiderivatives
5) Indefinite integrals by substitution, parts, partial fractions, trigonometric substitution, and using the table of integrals
6) Integrals for computing volumes of solids with known cross-sections: $V = \int_a^b A(x)\ dx$, where $A(x)$ is the cross-section area function. Volumes of solids of revolution are a special case of this.
7) Integrals for computing total mass/center of mass of wires and plates of given shapes, given the mass density function
8) Probability density functions of distributions, cumulative distribution function, median, mean.

9) Infinite series: geometric series; integral, comparison, ratio, and alternating series tests for convergence; power series, radius of convergence and endpoint behavior
10) Taylor polynomials and Taylor approximations, Taylor series by "shortcut rules" (substitution, algebraic manipulations, term-by-term integration and differentiation) for computing Taylor series from the known series for

$$e^x,\ \sin(x),\ \cos(x),\ (1+x)^p.$$

11) Differential equations
    a) What it means for a function to be a solution of a differential equation
    b) Graphical meaning of solutions via slope fields
    d) Formulas for solutions via separation of variables
    e) Applications to growth/decay problems.

Comments: The topics above that gave people the most difficulty on the midterm exams were 5 and 9. If you lost a lot of points on the midterms on those questions, start your review there!

Suggestions on How to Study

A) Reread your class notes and work through the examples we did in class – everything on the final will be very close to something we did along the way somewhere!
B) Look over your graded exams and the solutions, especially any questions you had difficulty with. Look over the solutions carefully – they're there to help you figure out what you did wrong and how to solve the problem correctly.
C) Also look at your graded problem sets and the write-ups from the group discussion days.
D) Look over the review sheets for the three full-period midterms.
E) Don't panic! There is a lot of material here, but not all that many key ideas. *Everything on the final will be very close to something we did along the way somewhere!*

The following are some sample exam-type problems taken from previous MATH 132 finals I have given. (Note: the actual final questions might be phrased and formatted differently or deal with additional topics from the list above. Also, the parts of the questions below all together are somewhat longer than the actual exam will be.)

I.

A) Give a precise statement of the Fundamental Theorem of Calculus.
B) The following graph shows $y = f(x)$. Let F be the antiderivative of f with $F(0) = 0$

and F continuous. Sketch the graph $y = F(x)$.

II. Compute each of the following integrals. If you use an entry from the table, say which one.

A) $\int 5x^4 - \sqrt{x} + \frac{3}{x}\ dx$

B) $\int x^3 \sin(x^4 + 1)\ dx$

C) $\int x^2 \sin(5x)\ dx$

D) $\int \frac{1}{4x^2+9}\ dx$

E) $\int \frac{(\ln(x))^4}{x}\ dx$

F) $\int (4 - x^2)^{3/2}\ dx$

G) $\int \frac{x+1}{x^3-4x}\ dx$

III. Let R be the region bounded by $y = x^{3/2}$, the x-axis, and $x = 0$, $x = 4$.

A) Find the arc length of the top border of R (the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$).

B) Find the volume of the solid obtained by rotating R about the x-axis.

C) Find the volume of the solid obtained by rotating R about the line $y = -4$.

D) A thin metal plate in the shape of R has mass density function $\rho(x) = x + 1$. What is the x-coordinate of the center of mass?

IV. At a particular location in Natick on the Mass Pike, a sensor was set up to measure the passage of traffic. The measurements made were used to derive a probability density function for the quantity x = time gap between successive cars. The results gave the following formula as a good fit for the density:

$$p(x) = \begin{cases} 0 & \text{if } x < 0 \\ .1e^{-.1x} & \text{if } x \geq 0 \end{cases}$$

A) Show that $p(x)$ satisfies the usual property for a probability density function:

$$\int_{-\infty}^{\infty} p(x)\ dx = 1.$$

B) What was the *median* time gap between successive cars?
C) What integral computes the *mean* time gap? Compute this mean.

V.
A) Use the integral test to determine whether the infinite series $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ converges or diverges.
B) Use the comparison test to determine whether the infinite series $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$ converges or diverges.
C) Use the ratio test to determine the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{x^n}{2^n n!}$

VI.
A) Using the definition of Taylor polynomials, compute the Taylor polynomial of degree $n = 3$ for $f(x) = \sin(x) + \cos(x)$ at $a = 0$. Use your answer from to approximate $f(.1)$ (.1 radians, of course!)

B) Use the error bound for Taylor polynomial approximations to determine an estimate of the number of decimal places of accuracy in your approximation in A.

C) Compare your approximate value for $f(.1)$ with a calculator value anc computing the true absolute error. How does this compare with your answer in B?

VII.
A) (10) Use our "shortcut methods" to find the first four nonzero terms in the Taylor series for $f(x) = \frac{1}{1+x^2}$ at $a = 0$.
B) (10) Use your answer from part A to derive the first four nonzero terms in the Taylor series for $g(x) = \arctan(x)$.
C) (5) Is the graph $y = \frac{\arctan(x)-x}{x^3}$ concave up or concave down at $x = 0$? How can you tell?

VIII. According to a simple physiological model, an athletic adult needs to consume food giving 20 calories per day per pound of body weight in order to maintain that body weight. If he or she consumes more or fewer calories, then *his or her body weight will change at a rate proportional to the difference between the number of calories needed to maintain body weight and the number of calories actually consumed.*

A) Say the person's calorie intake is constant at I calories per day. Write the statement in italics above as a differential equation for the body weight W as a function of t (time).
B) Find the general solution of your equation from part A using separation of variables.
C) The constant of proportionality in the relation above has been measured to be $\frac{-1}{3500}$. If a person starts out weighing 160lb and consumes 3000 calories per day, what will his weight be in one year (365 days)?