Mathematics 132 – Calculus for Physical and Life Sciences 2
Review Sheet for Midterm 3
April 20, 2005

General Information

The third midterm will be held the evening of Wednesday, April 27, from 6:00–7:30. Material from Section 8.6 through Section 10.4 will be covered. We will review for the exam in class on Tuesday and Wednesday next week.

Topics to be Covered

Here is a brief list of topics:

- Probability density (pdf) and cumulative distribution (cdf) functions; mean and median of a distribution.
- Numerical series: partial sums and convergence; finite and infinite geometric series. Convergence tests: integral, comparison, and ratio tests, alternating series.
- Power series, radius and interval of convergence; Taylor polynomials and series, series manipulations, the error estimate for Taylor polynomials.

Some Sample Problems

The following problems are typical of test-type questions. Note that a single problem may call for techniques spanning the coverage of the test. Do not attempt to memorize how to do "all the types of problems"; instead, strive to understand what each idea is used for, and select your approaches accordingly.

1. The following graph shows either a pdf or a cdf. Which type of function is it, and why? If it is a pdf, sketch the graph of the corresponding cdf; if it is a cdf, sketch the graph of the corresponding pdf.

2. Let x be the size of an email message in kilobytes (KB). An ISP finds that the fraction of emails between x and $x + \Delta x$ KB in size is about $cxe^{-0.01x}\Delta x$ (that is, the density function for x is $p(x) = cxe^{-0.01x}$).

   (a) Find the value of c

   (b) Calculate the fraction of email messages that are at most 100 KB in size,

(c) Calculate the fraction that are at least 50 KB in size.

(d) Calculate the median size of an email message.

(e) Give the integral that computes the mean size of an email message.

3. In a certain population, the height x of an individual in inches is normally distributed, with a mean of 68 and a standard deviation of 4. In other words, heights are modeled by the density function $p(x) = \frac{1}{4\sqrt{2\pi}}e^{-(x-68)^2/32}$.

   (a) Write down a definite integral whose value gives the fraction of the population whose height is between 68 and 72 inches, and perform the change of variables $z = (x-68)/4$ on your integral.

   (b) Write out the Taylor series for e^u giving both the first four non-zero terms and the summation form.

   (c) Use your answer from (b) to find the Taylor series for

   $$\int_0^x e^{-z^2/2}\,dz,$$

   giving both the first four non-zero terms and the summation form.

   (d) Calculate an estimate of the integral in part (a) using the series in part (c). (How?)

4. (a) Use the Comparison Test to determine whether or not

   $$\sum_{n=0}^{\infty} \frac{n+3^n}{2^n}$$

   converges.

   (b) Use the Integral Test to determine whether or not

   $$\sum_{k=0}^{\infty} \frac{k}{e^k}$$

   converges.

   (c) Use the Ratio Test to determine whether or not

   $$\sum_{k=0}^{\infty} \frac{3^n}{n!}$$

   converges.

5. Determine (with justification!) whether or not the following series converge:

   $$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}, \qquad \sum_{n=0}^{\infty} (-1)^n \frac{n^2+4n+1}{3n^4+2n^2+10000}, \qquad \sum_{n=1}^{\infty} \frac{1}{n^{1.01}}.$$

6. Let $f(x) = \sqrt{1+x} = (1+x)^{1/2}$. Find the 4th degree Taylor polynomial of f centered at $a = 0$. Find a factorial expression for the general term of the Taylor series.

7. Consider the geometric series $f(x) = \sum_{k=0}^{\infty} x^k = \dfrac{1}{1-x}$.

   (a) Use series manipulations to find the Taylor series of $xf'(x)$.

   (b) Use series manipulations to find the Taylor series of $-\ln(1-x)$.

   (c) Find the radius of convergence of the series in part (b), and investigate convergence at the endpoints.

   (d) Use parts (a) and (b) to *evaluate* the sums of the series $\sum_{k=1}^{\infty} \dfrac{1}{k \cdot 2^k}$ and $\sum_{k=1}^{\infty} \dfrac{k}{2^k}$.

8. For each of the given power series, find the interval of convergence.

$$f(x) = \sum_{n=1}^{\infty} \frac{(2x)^n}{\sqrt{n}}, \qquad g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-5)^n}{n \cdot 3^n}.$$

   (In particular, give the radius of convergence, and investigate convergence at the endpoints.)

9. The second degree Taylor polynomial of $f(x)$ at $a = 0$ is $p_2(x) = c + bx + ax^2$. What can you say about the signs of a, b, c if you know the graph of $f(x)$ is:

10. Use the error bound for Taylor approximations to estimate the number of decimal places of accuracy if the 6th degree Taylor polynomial at $a = 0$ is used to approximate $\cos(.8)$. Do the same for the nth degree polynomial in general. What happens to the error bound as $n \to \infty$?