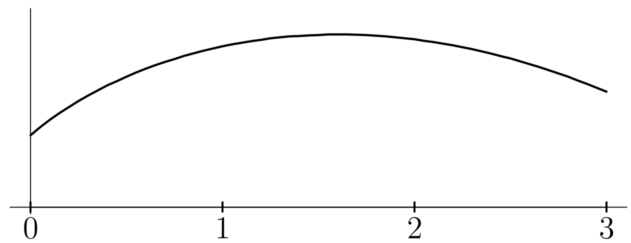
College of the Holy Cross, Spring Semester, 2005
Math 132 Answers to Review Sheet for Midterm 2

Practice Test

1. (a) For the function pictured below, LEFT(n), RIGHT(n), MID(n), and TRAP(n) approximations to $\int_0^1 f(x)\,dx$ yield the values 1.23, 1.34, 1.36, and 1.45 (in some order). Which method gave which value? Explain how you know which is which. (Note limits!)



Solution The function being integrated is increasing and concave down on the interval $[0, 1]$. Since f is increasing, LEFT(n) $<$ MID(n) $<$ RIGHT(n) and LEFT(n) $<$ TRAP(n). Because f is concave down, TRAP(n) $<$ MID(n). Thus

$$\text{LEFT}(n) = 1.23, \qquad \text{TRAP}(n) = 1.34, \qquad \text{MID}(n) = 1.36, \qquad \text{RIGHT}(n) = 1.45.$$

(b) Compute LEFT(2), RIGHT(2), MID(2), TRAP(2), and SIMP(2) for $\displaystyle\int_1^2 \frac{1}{x}\,dx$.

Solution LEFT(n) $= \frac{5}{6}$, RIGHT(n) $= \frac{7}{12}$, TRAP(n) $= \frac{17}{24} \simeq 0.70833$, MID($n$) $= \frac{24}{35}$, SIMP(n) $= \frac{1747}{2520} \simeq 0.693254$.

(c) Compute the exact value of the integral in (b), and the errors for the trapezoidal and Simpson's Rule approximations.

Solution ACTUAL$=\ln(2)$; TRAP error $\simeq -0.015186$, SIMP error $\simeq -0.0001068$.

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2. Determine whether each of the following integrals converges, and if so, find its value:

$$\int_1^\infty \frac{dx}{x^2+4} \text{ converges to } \tfrac{1}{2}\left(\tfrac{\pi}{2} - \arctan\tfrac{1}{2}\right), \qquad \int_0^5 \frac{du}{u^2-16} \text{ diverges.}$$

3. Consider the region R bounded by $y = 1 + x^2$, $x = 0$, $x = 1$, and the x-axis.

(a) Find the volume of the solid obtained by revolving R about the x-axis.

Solution $\pi \int_0^1 (1 + x^2)^2 \, dx = \frac{28}{15}\pi.$

(b) Find the volume of the solid obtained by revolving R about the line $y = -3$.

Solution $\pi \int_0^1 \left((4 + x^2)^2 - 3^2\right) dx = \frac{148}{15}\pi.$

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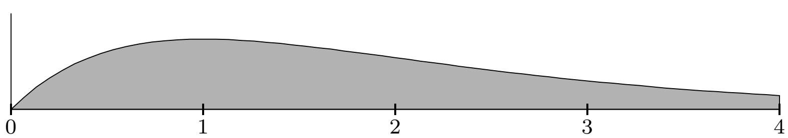
4. Let R be the region bounded by $y = xe^{-x}$, $y = 0$, $x = 0$ and $x = 4$.

(a) Sketch the region.



(b) A solid Ω has the region R as base and cross sections perpendicular to the x-axis that are squares. Find the volume of Ω.

Solution $\int_0^4 x^2 e^{-2x} \, dx = \frac{1}{4}(1 - 41e^{-8})$

(c) Set up, but do not evaluate, the integral to find the volume of the solid obtained if R is revolved about the x-axis.

(d) Set up, but do not evaluate, the integral to find the volume of the solid obtained if R is revolved about the line $y = -2$.

Solution (c) $\pi \int_0^4 x^2 e^{-2x} \, dx$, (d) $\pi \int_0^4 \left((xe^{-x} + 2)^2 - 2^2\right) dx.$

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5. A wire in the shape of the graph $y = x^2$, $-1 \le x \le 2$, has density $\delta(x) = 4 - x$ at the point (x, x^2).

(a) What is the arc length of the wire?

Solution $\int_{-1}^2 \sqrt{1 + 4x^2} \, dx = \sqrt{17} + \sqrt{\frac{5}{4}} + \frac{1}{4}\left(\ln\left|2 + \sqrt{\frac{17}{4}}\right| - \ln\left|-1 + \sqrt{\frac{5}{4}}\right|\right) \simeq 6.12573$

(b) Set up (with explanation) a Riemann sum approximating the *mass* of the wire.

(c) What definite integral computes the total mass?

(d) Evaluate the integral from part (c).

Solution The density of wire is nearly constant between x and $x + \Delta x$, and the length of the corresponding piece is roughly $\sqrt{1+4x^2}\,\Delta x$, so the mass of a small piece of wire is roughly $(4-x)\sqrt{1+4x^2}\,\Delta x$ grams, and the total mass is approximately

$$\begin{aligned}\sum(4-x)\sqrt{1+4x^2}\,\Delta x &\simeq \int_{-1}^{2}(4-x)\sqrt{1+4x^2}\,dx\\ &= \tfrac{31}{12}\sqrt{17}+\tfrac{29}{12}\sqrt{5}+\ln|2+\sqrt{\tfrac{17}{4}}|-\ln|-1+\sqrt{\tfrac{5}{4}}|\\ &\simeq 19.5935 \text{ grams}\end{aligned}$$

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6. A metal plate has the shape of the region in the plane bounded by $y = \sin x$, $y = 0$, $x = \pi/4$, and $x = \pi/2$. The density of the material is $\delta(x) = x$ grams per unit area.

(a) Compute the total mass of the plate.

Solution $\displaystyle\int_{\pi/4}^{\pi/2} x\sin x\,dx = 1 + \tfrac{\sqrt{2}}{2}(1-\tfrac{\pi}{4})$ grams

(b) Set up (but do not evaluate) the integral to compute the x-coordinate of the center of mass.

Solution $\displaystyle\int_{\pi/4}^{\pi/2} x^2\sin x\,dx \Big/ \int_{\pi/4}^{\pi/2} x\sin x\,dx$

(c) From physical intuition, should the y-coordinate of the center of mass of this plate be $\bar{y} = 1/2$? Why or why not?

Solution No, because at any given vertical distance from the centerline there is more mass below than above center.