Mathematics 132 – Calculus for Physical and Life Sciences 2
Exam 2 – Things To Know
March 21, 2005

General Information

As announced in the course syllabus, the second full-period midterm exam of the semester will be given the evening of Thursday, March 31. The format and schedule will be similar to that of the first midterm exam, and the midterms last semester.

Material To Know

The exam will cover the following topics we have studied since the first exam, including the following from Chapter 7, sections 5,6,7, and Chapter 8, sections 1,2,3. You will be provided with a copy of the table of integrals from the book. However, you should also carefully consider this *EXTREMELY IMPORTANT NOTE:* The material from section 7 of Chapter 7 and the sections in Chapter 8 depends heavily on the methods of integration covered on the first exam. You will also need to be prepared to carry out integration using any one of the methods there: u-substitution, parts, partial fractions, trigonometric substitution, and using the table of integrals. *Any one of those might appear on this exam too.* It would be good to start your review for this exam by reviewing the earlier sections of Chapter 7 also, especially if you feel shaky on those methods, or if you did not score well on the first exam.

1) *Numerical integration methods* (Left, Midpoint, Right sums, Trapezoidal Rule, and Simpson's Rule). Be prepared to carry out a calculation using any or all of these, for a given function on a given interval, with a small number of subintervals ($n \leq 4$). Questions 5,6,7 in Section 7.5 are good practice here! The most efficient ways to get the Trap and Simpson estimates are to use

$$Trap(n) = \frac{1}{2}(Left(n) + Right(n))$$

and

$$Simpson(n) = \frac{2Mid(n) + Trap(n)}{3}$$

(See Lab 1). Also know the geometry of these methods and be prepared to use it for questions like numbers 11-15 in Section 7.5.

2) Improper Integrals. Know what makes an integral improper (either an infinite interval, or an infinite discontinuity of the function), how to set up limits to evaluate one of these, and how to decide whether an improper integral converges or diverges. Some good practice problems here: Chapter 7 review: 127-140.
3) Setting up integral formulas by subdividing, approximating, summing, and taking limits. This is the basis for everything we did with volume, arclength, total mass, and center of mass problems.

4) Volumes of solids with known cross-sections (Cavalieri's Principle). Solids of revolution are a special case of this. (See 8.3/20-24, and Chapter 8 Review Problems 9-12)
5) Arclength of curves. Know the formula

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

for the arclength of $y = f(x)$, $x = a$ to $x = b$ and how to apply it. (See Chapter 8 Review Problems 7,8.)
6) Total mass and center of mass problems (Section 8.3). Know how to set up integrals computing these for rods or wires, and for thin plates with given shapes. (See Section 8.3/3,4,9,18-24.)

Practice Exam

I.
A) A function $y = f(x)$ has graph as in this diagram:

$Left(n), Right(n), Mid(n)$, and $Trap(n)$ approximations to $\int_0^1 f(x)\, dx$ yield the values $1.23, 1.34, 1.36, 1.45$ (in some order). Which method gave which value? Explain how you know which is which.
B) Compute $Left(2), Right(2), Mid(2), Trap(2)$, and $Simpson(2)$ approximations for

$$\int_1^2 \frac{1}{x}\, dx.$$

C) Compute the exact value of the integral in B, and the errors for the Trapezoidal and Simpson's Rule approximations.

II.
A) Does the integral

$$\int_1^\infty \frac{dx}{x^2 + 4}$$

converge? If so, what is its value?
B) Same question for

$$\int_0^5 \frac{du}{u^2 - 16}$$

III. Both parts of this problem refer to the region R bounded by $y = 1 + x^2$, $x = 0$, $x = 1$, and the x-axis.

A) Find the volume of the solid obtained by rotating R about the x-axis.
B) Find the volume of the solid obtained by rotating R about the line $y = -3$.

IV. Let R be the region bounded by $y = xe^{-x}$, $y = 0$, $x = 0$ and $x = 4$.

A) Sketch the region.
B) A solid has the region R as base and cross-sections perpendicular to the x-axis that are squares (extending the full width of the base). Find the volume.
C) Set up, but do not evaluate the integral to find the volume of the solid obtained if R is rotated about the x-axis.
D) Set up, but do not evaluate the integral to find the volume of the solid obtained if R is rotated about the line $y = -2$.

V. A wire in the shape of the graph $y = x^2$, $x \in [-1, 2]$ has density $\delta(x) = 4 - x$ at the point (x, x^2).
A) What is the total length of the wire? (Arc length!)
B) Set up a Riemann sum approximating the *total mass* of the wire, and explain how you got it.
C) What definite integral computes the total mass?
D) Evaluate your integral.

VI. A thin metal plate has the shape of the region in the plane bounded by $y = \sin(x)$, $y = 0$, $x = \pi/4$, and $x = \pi/2$. The density of the material of the plate at all points x units from the left-hand edge is $\delta(x) = x$ grams per unit area.

A) Compute the total mass of the plate.
B) Set up (but do not evaluate) the integral to compute the x-coordinate of the center of mass.
C) From physical intuition, should the y-coordinate of the center of mass of this plate be $\overline{y} = 1/2$? Why or why not?