Mathematics 132 – Calculus for Physical and Life Sciences 2
Exam 1 – Things To Know
February 16, 2005

General Information

As announced in the course syllabus, the first midterm exam of the semester will be given ("main seating") at 6:00 p.m. on Wednesday, February 23. You will have 90 minutes to work on the exam if you need that much time. The format will be similar to that of the midterm exams last semester. The exam will cover the material since the start of the semester, including the following material from Chapters 6, and sections 1-4 of Chapter 7.

1) The Fundamental Theorem of Calculus: If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

2) Antiderivatives graphically and numerically
3) Basic antiderivative rules: All rules coming from basic derivative formulas: Know $\int x^n\,dx$, $\int a^x\,dx$, $\int \sin(x)\,dx$, $\int \cos(x)\,dx$, $\int \frac{1}{x^2+1}\,dx$, $\int \frac{1}{\sqrt{1-x^2}}\,dx$, and so forth, plus the sum, and constant multiple rules
4) Solving differential equations $\frac{dy}{dx} = f(x)$ and applications to equations of motion problems
5) Integrals by substitution
6) Integrals by parts: Selecting appropriate u and dv, computing du and v, using the parts formula $\int u\,dv = uv - \int v\,du$, then finishing the integral on the right. Recall that this might involve using parts again, or another method such as substitution.
7) Integrals by the table: Recognizing the appropriate entry, using reduction formulas repeatedly if necessary. Be aware that a preliminary substitution might be necessary to take an integral to one of the forms in the table.
8) Integration of rational functions by partial fractions (see summary distributed in class).
9) Integration by trigonometric substitution (see summary distributed in class).

Note: Some problems may ask you to carry out a particular integration method on a problem. Others may leave the choice up to you. *Be prepared for both types of questions!* There will be a review for the exam in class on Tuesday, February 22. If necessary, we can also continue that on Wednesday, February 23 (the day of the exam).

Review Problems

The Review Problems at the ends of Chapter 6 and 7 are great for preparation for this exam. It's not necessary to work out every integral in Chapter 7 (there are over 100 of them!) But you should try a good selection and practice choosing a method at least for most of them.

Sample Exam

Note: This is somewhat longer than the actual exam will be, to give you an idea of the range of types of questions we might ask!

I. The following graph shows $y = F'(x)$, the *derivative* of F.

A) Assuming $F(0) = 0$, Compute $F(1), F(2), F(3), F(4)$ given the information in the graph.
B) Where are the critical points of F? Explain.
C) Sketch the graph $y = F(x)$ if $F(0) = 0$, and also if $F(0) = 2$.

III. A child throws a baseball vertically upward with velocity 12 ft/sec from an initial height of 4 feet. The acceleration of the baseball due to gravity is -32 ft/sec^2, so the velocity function v satisfies the differential equation

$$\frac{dv}{dt} = -32$$

A) Find formula for the velocity as a function of time.
B) When does the baseball reach its highest point? When does it hit the ground?
C) Find the greatest height the ball reaches.

IV.

A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\ dx$

B) Apply a u-substitution to compute $\int x(4x^2-3)^{3/5}\ dx$

C) Apply a u-substitution to compute $\int_1^2 e^{\sin(\pi x)} \cos(\pi x)\ dx$

D) Do you need partial fractions to compute

$$\int \frac{t^2+1}{t^3+3t+3}\ dt?$$

Explain, and give a simpler method.
E) Apply integration by parts to compute $\int x^2 e^{-2x}\ dx$

F) Apply partial fraction decomposition to compute

$$\int \frac{1}{x(x-1)(x+2)}\,dx$$

G) Which trigonometric substitution would you apply to compute $\int \frac{1}{u\sqrt{a^2-u^2}}\,du$? What trigonometric integral do you get after making the substitution? Which entry in the table applies to this form?

V. Compute each of the integrals below using some combination of basic rules, substitution, integration by parts, the table of integrals, partial fractions, and trigonometric substition. You must show all work for full credit.

A)

$$\int \frac{x^3}{x^2+4x}\,dx$$

B)

$$\int (x^2+2x)^{3/2}\,dx$$

C)

$$\int \frac{e^{\sqrt{\sin(x)}}\cos(x)}{\sqrt{\sin(x)}}\,dx$$

D)

$$\int \frac{dz}{z^2\sqrt{z^2+9}}$$