Mathematics 132 – Calculus for Physical and Life Sciences
Discussion 5 – Drug Levels
April 1, 2005

Background

The goal of today's discussion is to introduce some key ideas relating to *finite and infinite series*, the topic of Chapter 9 in the text which we are now ready to begin. We will do this in the context of an important question in clinical medicine. Namely, if a patient is taking regular doses of some drug over an extended period of time, *what is the long-term level of the drug that will build up in the body?* For some very toxic drugs, this might limit the dosage that could be given safely!

For instance, we will consider a drug called atenolol, which is usually given in 50mg doses once a day to lower blood pressure.

Discussion Questions

A) The half-life of atenolol in the bloodstream is about 6.3 hours. If a dose of $Q_0 = 50$ mg is given at the start of a 24 hour period, how much will be left after 24 hours have elapsed? Call this amount P_1, the amount left at the end of the first day.

B)

1) Next, a second dose is administered. For simplicity, let's assume it is absorbed into the bloodstream immediately. Call Q_1 the total amount present after the second dose (the new, plus the remnants of the first). Find Q_1. Similarly, let P_2 be the amount left at the end of the second day. Find P_2.
2) Next, at the start of the third day, a third dose is administered. Call Q_2 the total amount present after the third dose (the new, plus the remnants of the first and second). Find Q_2. Similarly, let P_3 be the amount left at the end of the third day day. Find P_3.

   (If you aren't starting to see a general pattern at this point, it may help to go a few days farther into the process.)

3) Say the patient is on a long-term treatment plan for high blood pressure. Continuing in the same way, find general formulas (using sums) for Q_n = amount present after the $(n+1)$st dose, and P_n = amount present at the end of the nth day.

C) Now, let's consider a general mathematical pattern. Let S_n be any sum of the form

$$S_n = a + ar + ar^2 + \cdots + ar^n \qquad (1)$$

where a, r are two constants. (This is called a *finite geometric series* with first term a and ratio r.) Let's see how to write S_n in "closed form" – that is, in an equivalent way but not including a summation of lots of terms.

1) What happens if you take $(1 - r)S_n = S_n - rS_n$ using the formula in (1)? Solve the resulting equation to give a formula for S_n in terms of a, r, n, but with out a summation.
2) Suppose $|a| < 1$. What happens in the limit as $n \to \infty$ to a^{n+1}? What does that say about $\lim_{n\to\infty} S_n$ (1)?

D) If we have a patient on a long-term drug regimen, say extending potentially over a period of many years, then we are talking effectively about letting $n =$ number of days go to ∞.

1) In the drug level problem in parts A and B, say in words what $\lim_{n\to\infty} Q_n$ and $\lim_{n\to\infty} P_n$ represent.
2) Suppose it is not safe for the patient ever to have an atenolol level > 70 mg. Is the treatment regimen described here safe? (Think about what your formula from part C says.)
3) Suppose we want there to be ≥ 10 mg of atenolol present in the body at all times (even just before a new dose is taken). Is the patient getting enough atenolol so this will be true "in the long run"? Explain.

Assignment

Group write-ups due in class on Tuesday, April 5.