Mathematics 132 – Calculus for Physical and Life Sciences 2
Discussion 3 – Choosing an Integration Method
February 16, 2005

Goals

We have now introduced a number of techniques for finding indefinite integrals of various types of functions and practiced each of them "in isolation". But of course, the ultimate goal here (and an important skill to have) is to *be able to recognize which method to apply* when you are given a "random, free-form" integral to do, and then be able to carry it out. So today, we will look at a collection of additional examples and practice deciding which method applies, then work out the integrals. Here's a good general strategy or "checklist" to follow as you decide:

- Can you do algebra on the integrand to make the function a sum of simple functions? (e.g. powers, exponentials, etc.)? If so, use the basic integration rules.
- If not, is the integral of the form $\int f(u)\,du$ for some function f that we have a basic integration formula for and some $u = u(x)$? (Note: It might take some imagination or algebra to see that you do have an integral of this form!) If so, do *substitution.*
- If the second point doesn't apply, is the function to be integrated a *rational function* (quotient of polynomials)? If so, do *partial fractions.* (Note: Some simple rational functions, including ones where the denominator has just one term won't require partial fractions, though.)
- If the third point doesn't apply, does the function contain $\sqrt{u^2+a^2}$, $\sqrt{u^2-a^2}$, or $\sqrt{a^2-u^2}$. If so, try the appropriate *trigonometric substitution.* In order for this to succeed, though, the the resulting trigonometric integral should be one you can do using the table.
- If the fourth point doesn't apply, can you identify a good choice for u, dv for *integration by parts*? (Of course some forms like $\int x^n e^x\,dx$, etc. can also be recognized as calling for parts immediately!)
- If none of the above apply, is there a substitution that puts the integral into a form where one does apply, or the table can be used?

For each integral below, say which method applies, and compute the indefinite integral. You will need to use the table of integrals in the text a couple of times.

Problems

A)
$$\int \frac{t^3}{\sqrt{16-t^2}}\,dt$$

(Note: This one can be done in several different ways!)

B)

$$\int \frac{x + x^{5/2}}{x^2}\, dx$$

C)

$$\int \frac{x \sin(\sqrt{x^2+3})}{\sqrt{x^2+3}}\, dx$$

(Hint: This is an easy one if you see the right way. Don't be fooled by the square roots of quadratics.)

D)

$$\int v^2 \arcsin(v)\, dv$$

E)

$$\int_0^{\sqrt{\pi/2}} x(\sin(x^2))^5 \cos(x^2)\, dx$$

F)

$$\int \sqrt{x^2 - 4x}\, dx$$

(Hint: complete square first.)

G)

$$\int \frac{u+1}{u^3 + 8u^2 + 17u}\, dx$$

Assignment

Group writeups due Monday, February 21.