Mathematics 244, section 1 – Linear Algebra
Sample Exam 1 – February 9, 2007

Directions

There are 100 points.

I. Let S be a subset of $\mathbf{R}^n$.

A) (5) Give the definition of Span(S), the *span* of S.

B) (15) Show that S is linearly dependent if and only if there exists some $\mathbf{x} \in S$ which satisfies $\mathbf{x} \in \text{Span}(S \setminus \{\mathbf{x}\})$.

II. (15) Is

$$\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \right\}$$

a linearly independent subset of $\mathbf{R}^3$? Why or why not?

III. (15) Put the augmented matrix of this system of linear equations into row-reduced echelon form and find a parametrization for the set of solutions:

$$\begin{array}{ccccccccc} 2x_1 & & & + & x_3 & - & x_4 & = & 0 \\ x_1 & + & 3x_2 & & & + & 2x_4 & = & 1 \\ & & x_2 & + & x_3 & + & x_4 & = & -1 \end{array}.$$

IV.

A) (5) Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the linear mapping with $T(e_1) = 5e_1 + e_3$, $T(e_2) = 2e_2 - e_3$, $T(e_3) = e_1 - e_2$. What is the matrix A such that $T = T_A$?

B) (15) Is the matrix A you found in part A invertible? If so, find A^{-1}. If not, give a reason why you know it is not invertible.

V. (True-False) Determine whether each of the following statements is true or false. For the ones that are true, give short proofs; for those that are false, give counterexamples.

A) (10) Let A be an $n \times n$ matrix. If the system of linear equations $Ax = b$ has just one solution x for some $b \in \mathbf{R}^n$, then it has just one solution for all b.

B) (10) If $W = \text{Span}(\{\mathbf{v}, \mathbf{w}\})$ then for any scalar $a \neq 0$, $W = \text{Span}(\{a\mathbf{v} + \mathbf{w}, \mathbf{w}\})$ also.

C) (10) If a system of linear equations has more variables than unknowns, there is automatically a nonzero solution.