Mathematics 244, section 1 – Linear Algebra
Practice True-False Questions for Exam 2
March 19, 2004

For each true statement, give a short proof or example; for each false statement give a reason or a counterexample

A) If $T : V \to W$ is linear, $\dim(V) = 5$, $\dim(W) = 7$ then T is surjective.

B) Every linear $T : V \to W$, where $\dim(V) = 5$, $\dim(W) = 7$, is injective.

C) Some linear $T : V \to W$, where $\dim(V) = 5$, $\dim(W) = 7$, is injective.

D) If $\{v_1, v_2, \ldots, v_n\}$ in V is linearly dependent, and $T : V \to W$ is linear, then

$$\{T(v_1), \ldots, T(v_n)\}$$

in W is linearly dependent.

E) If $T : \mathbf{R}^n \to \mathbf{R}^n$ is linear, then $Ker(T) \cap Im(T) = \{\vec{0}\}$.

F) If $T : V \to V$ is linear, and $\beta = \{v_1, \ldots, v_n\}$, $B' = \{cv_1, \ldots, cv_n\}$, $c \neq 0 \in \mathbf{R}$ are bases for V, then the matrices A, A' of T with respect to β, β and β', β' are equal.

G) If $\mathbf{A}$ is an $n \times n$ matrix, and c is a fixed real number, the set $W = \{v \in \mathbf{R}^n : Av = cv\}$ is a vector subspace of $\mathbf{R}^n$.

H) If $T = I_{\mathbf{R}^2}$ is the identity mapping and β, β' are two bases for $\mathbf{R}^2$, then the matrix of T with respect to β, β' is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

I) If $T : V \to W$ is an isomorphism, and $U \subseteq V$ is a vector subspace, then $\dim(U) = \dim(T(U))$.

J) If $v_1, \ldots, v_n$ are vectors in a vector space V, and $w_1, \ldots, w_n$ are vectors in a vector space W, then there always exists some linear $T : V \to W$ satisfying $T(v_i) = w_i$ for all $i = 1, \ldots, n$.

K) If $T : \mathbf{R}^n \to \mathbf{R}^n$ is linear, and $Ker(T) + Im(T) = \mathbf{R}^n$, then $Ker(T) \cap Im(T) = \{\vec{0}\}$.