Mathematics 244 – Linear Algebra
Discussion 1 – More Examples of Subspaces
January 28, 2004

Background

Recall that we have said a subset W of a vector space V is a (*vector*) *subspace* if W is itself a vector space under the vector sum and scalar multiplication operations from V. We also saw the following theorem that gives a "shortcut test" for deciding whether a given $W \subseteq V$ is a subspace:

Theorem. Let V be a vector space. A nonempty subset $W \subset V$ is a subspace if and only if whenever $\mathbf{x}, \mathbf{y} \in W$ and $c \in \mathbf{R}$, then $c\mathbf{x} + \mathbf{y} \in W$ (using the vector sum and scalar multiplication operations from V).

Today, in our first discussion, we will get some additional practice using the Theorem to decide whether given subsets are subspaces.

Discussion Questions

A) For each of the given vector spaces V and subsets $W \subset V$, using the test given by the theorem above, decide whether or not W is a subspace of V.

1) $V = \mathbf{R}^2$ $W = \{(x, y) : y = 12x^2\}$
2) $V = \mathbf{R}^3$, $W = \{(x, y, z) : 2x + 3y - z = 0 \text{ and } x - y + z = 0\}$. Also, what is the set W in geometric terms?
3) $V = \mathbf{R}^4$, $W = \{(x, y, z, w) : x - y + 2z + w = 1\}$
4) $V = F(\mathbf{R})$ (that is, the vector space of all functions $f : \mathbf{R} \to \mathbf{R}$ and
$$W = \{f \in F(\mathbf{R}) : f(x+1) = f(x) \text{ for all } x\}.$$
Also, give an example of a nonconstant function in W.
5) $V = P_5(\mathbf{R})$ (the vector space of polynomials of degree ≤ 5 with real coefficients), $W = \{p \in V : x^2 p''(x) = 3xp'(x) - 3p(x)\}$. Also find an example of a nonzero polynomial in W. Hint: A good place to start is to determine for which exponents n we have $x^n \in W$.

B) We saw in class that the *intersection* of any collection of subspaces of a vector space is a subspace. Is the same true for *unions* of subspaces? (Recall $W_1 \cup W_2 = \{\mathbf{x} : \mathbf{x} \in W_1 \text{ or } \mathbf{x} \in W_2\}$.) *Suggestion:* Look at some examples in $V = \mathbf{R}^3$ or $\mathbf{R}^2$.

Assignment

Group writeups due in class, Monday, February 2.