Mathematics 244 – Linear Algebra

Additional Review True-False Questions – Exam 3

April 23, 2004

For the true statements, give a short proof. For the false statements, give a reason or a counterexample.

A) If A is a 3×3 matrix whose characteristic polynomial has a double root, then A is not diagonalizable.

B) If $T : \mathbf{R}^2 \to \mathbf{R}^2$ and there is a non-zero vector x that satisfies $T^2(x) = 4x$, then $T(x) = \pm 2x$.

C) Every 3×3 matrix has at least one eigenvalue $\lambda \in \mathbf{R}$.

D) If λ is an eigenvalue of $(A - sI)^{-1}$, where $s \in \mathbf{R}$, then $s + \frac{1}{\lambda}$ is an eigenvalue of A.

E) The eigenvalues of $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$ are $\lambda = a + b$ and $\lambda = a - b$.

F) If the linear mapping $T : \mathbf{R}^3 \to \mathbf{R}^3$ has eigenvalues $\lambda = 1, 2$, and x_1, x_2 are linearly independent eigenvectors for $\lambda = 1$, while y is an eigenvector for $\lambda = 2$, then $E = \{x_1, x_2, y\}$ is linearly independent.

G) If A is an $n \times n$ matrix with linearly independent rows, then $\det(A) = 0$.

H) Let V be a vector space of dimension n and $T : V \to V$ be linear. If $[T]_\alpha^\alpha = I_n$ (the $n \times n$ identity matrix) with respect to *some* basis α for V, then T is the identity mapping.