Mathematics 400–Directed Readings in Algebra
Course Plan–Spring 2020

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Course Description

We plan to cover the same material as (one common version of) the Modern Algebra 2 course when there is sufficient demand and staffing for that to be offered. This will fill a gap in John Graf's preparation for pure mathematics graduate programs and give him some additional background.

We will meet twice a week for 75-minute sessions (exact schedule to be determined). Each meeting will involve some presentations of problem solutions by John, some introduction of new material, by me and possibly by John on occasion.

The course will concentrate on the theory of *rings and fields*, an area of abstract algebra that grew historically out of the desire to understand the basic algebraic properties of arithmetic operations such as factorization of integers and its generalizations, and the solution of algebraic equations. Our main goal is a fascinating subject called "Galois Theory" (after the unfortunate 19^{th} century French mathematician Évariste Galois). This theory gives a complete answer to the question whether a polynomial equation

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0 = 0$$

can be solved by means of an radical formula in the coefficients (such as the *quadratic formula* in the case $n = 2$). Galois Theory associates to such an equation a certain group of permutations of the roots of the equation, whose algebraic structure determines whether a "radical" formula for the roots exists.

The topics we will be studying are:

1. The basic language of ring theory, ideals and homomorphisms
2. Factorization in rings, unique and otherwise
3. Fields, algebraic and transcendental extensions, applications to the ancient Greek construction problems (duplication of the cube, squaring the circle, trisecting a general angle).
4. Galois Theory of equations

We will spend roughly equal amounts of time on each unit.

Text

We will use a standard advanced undergraduate algebra text such as Artin, Herstein, or possibly the even more advanced text by Dummit and Foote (used in many 1st year graduate courses).

Assignments and Grading

The grading will be based on

1. Weekly problem sets (40%)
2. Problem presentations (10%)
3. Larger midterm problem set (20%)
4. Larger comprehensive final problem set (30%)