Mathematics 134 – Intensive Calculus for Science 2
Final Examination – May 10, 2006

Directions: Do all work in the blue exam booklet. There are 200 regular points and 20 Extra Credit points, distributed as marked. Take a deep breath before plunging in and read each question carefully.

I.

A) (10) Give a precise statement of the Fundamental Theorem of Calculus (both parts).

B) (15) The following graph shows $y = f(x)$. Let F be the antiderivative of f with $F(0) = 0$ and F continuous. Sketch the graph $y = F(x)$.

II. Compute each of the following integrals. You may use the table of integrals anywhere on these. If you do, say which table entry you are using.

A) (10) $\int 3x^6 - 4\sqrt{x} + \sin(x)\ dx$

B) (10) $\int \frac{x^3}{x^4+1}\ dx$

C) (15) $\int x^2 e^{-9x}\ dx$

D) (10) $\int \frac{1}{\sqrt{4x^2+9}}\ dx$

E) (15) $\int \frac{x+1}{x^2+5x+6}\ dx$

III. Let R be the region bounded by $y = x$, the x-axis, and $x = 1$, $x = 4$.

A) (15) Find the volume of the solid obtained by rotating R about the line $y = -2$.

B) (10) A thin plate has the shape of the region R (x, y in cm) and density $\delta(x) = x^{-1/2}$ grams/cm^2. Find its total mass.

IV. At a particular location in Natick on the Mass Pike, a sensor was set up to measure the passage of traffic. The measurements made were used to derive a probability density function for the quantity $x =$ time gap between successive cars (in minutes). The results gave the following formula as a good fit for the pdf: $p(x) = 11(1-x)^{10}$ if $0 < x < 1$, and zero otherwise.

A) (10) Show that p satisfies the usual property for a probability density function: $\int_0^1 p(x)\ dx = 1$.

B) (15) What is the probability that the time gap between successive cars is between $x = .1$ minute and $x = .2$ minute?

V.

A) (10) Using the definition of Taylor polynomials, compute the Taylor polynomial of degree $n = 3$ for $f(x) = \sqrt{1 + 2x}$ at $a = 0$.

B) (10) Use our shortcut methods to check your work in part A.

C) (5) Use your polynomial from part A to compute an approximation to $\sqrt{1.2}$. What is the error in your approximation?

VI. (15) Solve for y by separation of variables: $\frac{dy}{dx} = \cos(x)(1 + y^2)$ with $y(0) = 1$.

VII. An avian flu epidemic has broken out in Birdsburgh, a large city with total population 10 million. Write N for the number of people who have been infected, as a function of time. The Birdsburgh Public Health department determines that:

$$\frac{dN}{dt} = kN(10 - N), \tag{1}$$

or in words: the rate of change of N is proportional to the product of N and the number of people not yet infected, where N is in millions of people, t in weeks, k a positive constant.

A) (10) Which of the following slope field plots matches (1)? Explain how you can tell.

B) (10) For what value of k is $N(t) = 10/(1 + 1000e^{-t})$ a solution of (1)?

C) (5) If the epidemic proceeds according to the function $N(t)$ in part B, how many weeks will pass before the number of infected people reaches 1 million?

Extra Credit (20) The hull of a boat is 20 feet long. At a distance s feet from the bow (the front), the cross section of the part of the hull below $y = 0$ (the waterline) has the shape of the region in the xy-plane below $y = 0$ and above the parabola $y = ax^2 - b$, where a, b are given in the following table:

s	5	10	15	20
a	2	3	4	5
b	2	3	4	4

Estimate the *volume* enclosed by the hull below the water line.

Have an enjoyable, safe, and productive summer!