Mathematics 133 – Intensive Calculus for Science 1
Sample Exam III – November 22, 2005

I. Find derivatives of each of the following functions. Any correct form of the derivative is OK here.

A) $f(x) = \ln(x) + 2^{4x}$

B) $g(x) = \sin(x)\cos(x^2)$

C) $h(x) = \arctan(x^2 + e^{2x} + 1)$

D) Find $\frac{dy}{dx}$ if $x^2y^2 - 2xy + x^3 = 1$.

II. All parts of this question refer to the function

$$f(x) = \frac{x}{x^2 + 4}$$

A) Compute $f'(x)$. Simplify and factor the numerator.

B) Using your $f'(x)$, find all critical points of f and classify each as a local maximum, local minimum, or neither using the *First Derivative Test*.

C) Compute $f''(x)$. Simplify and factor the numerator again.

D) Using your $f''(x)$, at which x are the inflection points on the graph $y = f(x)$ located?

III. The hypotenuse of a right triangle has one end at $(0, 0)$ and the other at a point on $y = x^2e^{-3x}$ with $x \geq 0$. One of the other two sides lies along the x-axis and the other is parallel to the y-axis. Find the maximum area of such a triangle.

IV. A chemical storage tank is an "upside-down" cone. The depth is 12 meters and the top radius is 5 meters. When the depth of the chemical in the tank is 1 meter the level is falling at the rate of 1 meter per minute. How fast is the volume of chemical changing then?