

I. The following graph shows *the derivative*  $y = f'(x)$  for some function  $f(x)$ .

- A) (15) Using the information here, construct a “qualitative” plot of  $y = f''(x)$ .
- B) (10) Over which intervals is  $f$  increasing?
- C) (5) Is  $f'$  continuous at  $x = 1$ ? Why or why not? What happens on the graph  $y = f(x)$  at  $x = 1$ ?

II.

- A) (10) The function  $H(t)$  gives the number of hours of daylight  $t$  days after the start of the year in Worcester. At  $t = 304$  days (October 31 in a non-leap year),  $H'(304) = -0.083$ . Give the meaning of this equation as a sentence, using appropriate units.
- B) (10) The table below gives the position  $s$  (in miles) of a freight train moving along a straight line track as a function of time  $t$  (in hours).

$t$	.5	1	1.5	2	2.5
$s$	10	25	42	50	55

Estimate the train’s instantaneous velocity at  $t = 1.5$  hours as closely as you can from this information.

III. (15) Using the limit definition, find  $f'(x)$  for  $f(x) = 1/x$ .

IV. Find derivatives of each of the following functions by applying the appropriate “short-cut” derivative rules:

- A) (10)  $f(x) = 5x^7 - \frac{3}{\sqrt{x}} - 4^{2x}$
- B) (10)  $g(x) = (x^2 + 1)^{12}2^x$
- C) (10)  $h(x) = \frac{x^2}{e^x - 1}$

V. (5) Say whether the following statement is true or false, and explain your reasoning: If the time interval is short enough, then we expect the average velocity of a car over the interval will be close to its instantaneous velocity at any time in the interval.