Mathematics 133 – Intensive Calculus for Science 1
"Quiz 0" – Algebra Review September 2, 2005

Your Name: *Solutions – –AnswerKey*

Do all work on this sheet (front and back). You have 20 minutes. **No calculators on this quiz.**

A) Write as a single quotient and simplify: $\frac{\frac{1}{x}}{3-x^2} + \frac{2-x}{3+x^2}$

To add fractions, we find a common denominator, then add the numerators:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

Here the result is

$$\begin{aligned}\frac{\frac{1}{x}}{3-x^2} + \frac{2-x}{3+x^2} &= \frac{\frac{1}{x}(3+x^2)}{(3-x^2)(3+x^2)} + \frac{(2-x)(3-x^2)}{(3+x^2)(3-x^2)} \\ &= \frac{\frac{3}{x} + x + 6 - 3x - 2x^2 + x^3}{9-x^4} \\ &= \frac{3 + 6x - 2x^2 - 2x^3 + x^4}{9x - x^5}\end{aligned}$$

It's also OK to leave the denominator in factored form: $x(3-x^2)(3+x^2)$.

B) Solve for t: $5(t-2)+1 = 12-t$.

Multiply out on the left: $5t - 10 + 1 = 12 - t$ then add $t + 9$ to both sides to put all the terms with t on the left: $6t = 21$. Hence $t = 21/6 = 7/2$.

C) Solve for x: $2x^2 - 3x = 3$ (find all solutions)

The left side of the equation $2x^2 - 3x - 3 = 0$ does not factor easily, so we use the *Quadratic Formula*: The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Here $a = 2, b = -3, c = -3$, so

$$x = \frac{3 \pm \sqrt{9+24}}{4} = \frac{3 \pm \sqrt{33}}{4}$$

D) Solve for a: $27a^{1/3} = 3a$

First method: Cube both sides to get rid of the cube root $a^{1/3} = \sqrt[3]{a}$: $27^3 a = 27a^3$, or $a^3 - 27^2 a = 0$. This factors as $a(a-27)(a+27) = 0$, so $a = 0$, $a = 27$, or $a = -27$.

Second method: Notice that $a = 0$ is one solution. If $a \neq 0$, then we can divide both sides by $3a^{1/3}$ to get $9 = a^{2/3}$. Hence $a = 9^{3/2} = (\pm\sqrt{9})^3 = (\pm 3)^3 = \pm 27$.

E) Simplify: $\left((1+c)^{3/2}\right)^4 \cdot (1+c)^{-2}$

By the rules for exponents:

$$\left((1+c)^{3/2}\right)^4 \cdot (1+c)^{-2} = (1+c)^{4\cdot 3/2}(1+c)^{-2} = (1+c)^{6-2} = (1+c)^4.$$

F) Expand and simplify: $(a^2 - 2b + 2)(2b - a + 1)$

Answer: $2a^2b - 4b^2 + 4b - a^3 + 2ab - 2a + a^2 - 2b + 2 = 2a^2b - 4b^2 + 2b - a^3 + 2ab - 2a + a^2 + 2$.

G) Find the coordinates of the center and the radius of the circle with equation $x^2 - 6x + y^2 + 4y = 0$.

Answer: Completing the square in x and y, we find: $(x-3)^2 - 9 + (y+2)^2 - 4 = 0$, or $(x-3)^2 + (y+2)^2 = 13$. The center is at $(3, -2)$ and the radius is $\sqrt{13}$.