Mathematics 133 – Intensive Calculus for Science 1
Precalculus Review Problems
August 31, 2005

The following problems will not be collected or graded. They are similar to questions on the first diagnostic quiz to be given in class on Friday, August 31.

1. Find all values of x that satisfy the given inequality or inequalities:

   a) $-4x \geq 20$ – *Answer:* all $x \leq -5$.
   b) $x + 1 > 4$, or $x + 2 < -1$ – *Answer:* all $x > 4$ or $x < -3$ (could also be written as a union of intervals: $(-\infty, -3) \cup (4, +\infty)$)
   c) $x + 3 > 1$ and $x - 2 < 1$ – *Answer:* all $x > -2$ and $x < 3$ (could also be written as the interval $(-2, 3)$.)

2.

   a) Rewrite using positive exponents only: $\frac{x^{-1/3}}{x^{1/2}}$ – *Answer:* $\frac{1}{x^{5/6}}$
   b) Simplify: $(x^2y^{-3})(x^{-5}y^3)$ – *Answer:* $x^{-3} = \frac{1}{x^3}$
   c) Simplify: $\left(\frac{x^3}{-27y^{-6}}\right)^{-2/3}$ – *Answer:* $\frac{9}{x^2y^4}$
   d) Simplify: $\left(\frac{x^{-3}}{y^{-2}}\right)^2 \left(\frac{y}{x}\right)^4$ – *Answer:* $\frac{y^8}{x^{10}}$

3. A salesperson's monthly commission is 15% on all sales over \$12000. If the goal is to make a commission of at least \$3000 per month, what monthly sales figure should he or she attain? – *Answer:* Call the monthly sales figure x. (Assuming no commission on the sales under 12000), we want $(.15)(x - 12000) \geq 3000$ so $x \geq 32000$.

4. The diameter x in inches of a batch of ball bearings manufactured by PAR Mfg. satisfies the inequality $|x - .1| \leq .001$. What are the largest and smallest diameters a ball bearing in the batch can have? – *Answer:* $.099 \leq x \leq .101$.

5. Perform the indicated operations and simplify:

   a) $x - (2x - (-x - (1 - x)))$ – *Answer:* $-x - 1 = -(x + 1)$
   b) $2(t + \sqrt{t})^2 - 2t^2$ – *Answer:* $2t(2\sqrt{t} + 1)$ (or $4t^{3/2} + 2t$)
   c) $(2x^2 - 1)(x) - x^2(x + 2)$ – *Answer:* $x^3 - 2x^2 - x$
   d) $5x^2(3x + 1)^4(6x) + (3x + 1)^5(2x)$ – *Answer:* The "simplest" form is probably the factored form: $2x(3x + 1)^4(15x^2 + 2x + 1)$. If you expand and collect terms, you should get: $2430x^7 + 3726x^6 + 2430x^5 + 900x^4 + 210x^3 + 30x^2 + 2x$

6. Factor out the greatest common factor:

   a) $7a^4 - 42a^2b^2 + 49a^3b$ – *Answer:* $7a^2(a^2 - 6b^2 + 7ab)$
   b) $xe^{-2x} - x^3e^{-x}$ – *Answer:* $xe^{-x}(e^{-x} - x^2)$

7. Factor:

   a. $9x^2 - 16y^4$ – *Answer:* $(3x - 4y^2)(3x + 4y^2)$ (difference of squares!)
   b. $3x^2 - 6x - 24$ – *Answer:* $(3x + 6)(x - 4)$
   c. $6ac + 3bc - 4ad - 2bd$ – *Answer:* $(2a + b)(3c - 2d)$

8. Solve for x:

   a. $x^2 + x - 12 = 0$ – *Answer:* by factoring (or quadratic formula): $x = 3, -4$
   b. $4x^3 + 2x^2 - 2x = 0$ – *Answer:* by factoring: $x = 0, 1, -1/2$
   c. $8x^2 - 8x - 3 = 0$ – *Answer:* by quadratic formula: $x = \frac{2 \pm \sqrt{10}}{4}$

9. Simplify:

   a. $\frac{2a^2 - 2b^2}{b - a} \cdot \frac{4a + 4b}{a^2 + 2ab + b^2}$ – *Answer:* -8 (factor and cancel!)
   b. $\frac{58}{3(3+t^2)} + \frac{1}{3}$ – *Answer:* $\frac{t^2 + 61}{3(3+t^2)}$
   c. $\frac{2x}{2x-1} - \frac{3x}{2x+5}$ – *Answer:* $\frac{-2x^2 + 13x}{(2x-1)(2x+5)}$
   d. $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x^2}}$ – *Answer:* $\frac{x}{x-1}$
   e. $\frac{2x(x+1)^{-1/2} - (x+1)^{1/2}}{x^2}$ – *Answer:* $\frac{x-1}{x^2\sqrt{x+1}}$

10.

   a. On a set of coordinate axes, plot the points $P = (1, 3)$ and $Q = (4, 7)$. Determine the distance $d(P, Q)$ between them. – *Answer:* distance is 5.
   b. What is the Cartesian equation of the circle with center $(h, k) = (4, 2)$ and radius $r = 6$. – *Answer:* $(x - 4)^2 + (y - 2)^2 = 36$, or $x^2 - 8x + y^2 - 4y = 16$
   c. The equation $x^2 + y^2 + x - 6y = 0$ defines a circle in the plane. Find its center and radius. – *Answer:* Completing the square: $(x + 1/2)^2 + (y - 3)^2 = 37/4$ center: $(-1/2, 3)$, radius $\sqrt{37}/2$
   d. Do the points $(3, 4), (-3, 7), (-6, 1), (0, -2)$ form the vertices of a square in the plane? Why or why not? – *Answer:* Yes – Call the points $P = (3, 4), Q = (-3, 7), R = (-6, 1), S = (0, -2)$. Then the distance from P to Q is

   $$\sqrt{(3 + 3)^2 + (4 - 7)^2} = \sqrt{45} = 3\sqrt{5}.$$

   The distances from Q to R, from R to S, and S to P work out to be the same, so $PQRS$ is a parallelogram with 4 equal sides (a rhombus or "diamond"). Then, the line from P to Q has slope $\frac{7-4}{-3-3} = \frac{-1}{2}$, while the line from Q to R has slope $\frac{1-7}{-6+3} = 2$. This shows that $PQ \perp QR$, so the rhombus is a square.