Mathematics 133–Intensive Calculus for Science 1
Solutions for Exam 2
November 4, 2005

I.

A) From the graph, $f(x)$ appears to be differentiable everywhere except $x = 0$, where there is a sharp corner.

B) The derivative $y = f'(x)$:

Note: $f'(x) > 0$ is positive on intervals where $f(x)$ is increasing and negative on intervals where $f(x)$ is decreasing. At $x = 0$, there is a jump discontinuity on the graph $y = f'(x)$ because of the sharp corner on the graph $y = f(x)$.

II.

$$\begin{aligned} f'(x) &= \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h\to 0} \frac{(x+h)^2 - (x+h) + 4 - x^2 + x - 4}{h} \\ &= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x - h + 4 - x^2 + x - 4}{h} \\ &= \lim_{h\to 0} \frac{2xh + h^2 - h}{h} \\ &= \lim_{h\to 0} 2x + h - 1 \\ &= 2x - 1 \end{aligned}$$

III.

A) (Product, exponential, and power rules)

$$f'(x) = 2^x \ln(2)(x^4+3x^{-1/3})+2^x(4x^3-x^{-4/3}) = 2^x(\ln(2)x^4+3\ln(2)x^{-1/3}+4x^3-x^{-4/3})$$

B) By the chain rule (twice):

$$g'(x) = 3(e^{\pi x} + 2)^2 \pi e^{\pi x}$$

C) By the quotient rule:

$$h'(x) = \frac{(2x^2 + 3x)(2x) - (x^2 - 3)(4x + 3)}{(2x^2 + 3)^2} = \frac{3x^2 + 12x + 9}{(2x^2 + 3)^2}$$

IV. The price in 2000 was $P(50) = 100(1.05)^5 0 \doteq \$1146.74$. The price was changing at $P'(50) = 100(1.05)^5 0 \ln(1.05) \doteq 55.95$ dollars per year. It was increasing because $P'(50) > 0$.

V. $C'(q) = \frac{50}{4} q^{-3/4}$, so $C''(q) = \frac{-150}{16} q^{-7/4}$. The cost function graph is concave down for $q > 0$ since $C''(q) < 0$. In everyday terms, this means that as the number of copies is increased, the *cost per copy decreases.*