Mathematics 133 – Intensive Calculus for Science 1
Discussion 1 – Linear and Exponential Functions, continued
September 7, 2005

Working in a Group

Since this is the first of the discussion class of the semester, a few words about this way of working are probably in order. In the discussion meetings of this class, we will be aiming for *collaborative learning* – that is, for an integrated group effort in analyzing and attacking the discussion questions where everyone in each of the groups is fully involved in the process. The idea is that, by actively participating in the class through talking about the ideas yourself in your own words, you can come to a better first understanding of what is going on than if you simply listen to someone else (even me!) talk about it. However, to get the most out of this kind of work, some of you may have to adjust some of your preconceptions. In particular:

- This is *not a competition*. You and your fellow group members are working as a team, and the goal is to have everyone understand what the group does fully.
- At different times, it is inevitable that different people within the group will have a more complete grasp of what you are working on and others will have a less complete grasp. Dealing with this a group setting is excellent preparation for real work in a team (the way many desirable jobs in the real world are structured); it also offers opportunities for significant educational experiences:
- If you feel totally "clueless" at some point and everyone else seems to be "getting it," your job will be to ask questions and get your fellow group members to explain what they are saying until you are fully satisfied. (Don't forget, the others may be jumping to unwarranted conclusions, and your questions may save the group from pursuing an erroneous train of thought!)
- On the other hand, when you think you do see something, you need to be willing to explain it patiently to others. (Don't forget, the absolutely best way to make sure you *really understand* something is to be able to explain it to someone else. If you are skipping over an important point in your thinking, it can become very apparent when you set out to convey your ideas to a team member.)

In short, *everyone has something to contribute, and everyone will contribute in different ways at different times.*

Background

We have seen that linear functions can be described by equations $y = mx + b$ or $y = m(x - x_1) + y_1$. Exponential functions, on the other hand look like $y = ba^x$. In the first question, we will explore some of the information that is contained in the equation, and how that relates to the graphs of linear functions. Finally question D deals with the difference between exponential and linear functions. (Turn over for questions).

Discussion Questions

A) (No graphing calculators on this question!) Do problem 11 from Section 1.1 (page 7) of the text. Give full explanations of how you are matching the formulas with the graphs, in one or more complete sentences.

B) Recall that saying a quantity y is *(directly) proportional* to another quantity x is the same as saying $y = kx$ for some constant k.

1) Explain why saying that y is *(directly) proportional* to quantity x says that y is a linear function of x.
2) For example, the circumference of a circle is proportional to the radius of the circle. What is the constant of proportionality in that case?
3) Write as a formula: "the energy E expended by a swimming dolphin is proportional to the cube of its speed v."

C) One potentially tricky point in section 1.2 in our text is the distinction between growth rates and *continuous* growth rates. Here's the "run-down": Saying P grows or decays at a rate of $r\%$ per time unit means $P(t) = P_0(1 + r/100)^t$. $r > 0$ means growth; $r < 0$ means decay. (For instance in our city example from class on Tuesday, where growth rate was 2% per year we had $P(t) = 10(1.02)^t$.) *On the other hand*, saying P grows or decays at a *continuous* rate of k means P is written in terms of the "natural exponential" function as $P(t) = P_0e^{kt}$. For instance, $k = .10$ might be described as a 10% continuous growth rate. Similarly $k = -.05$ might be described as a 5% continuous decay rate.

1) Write the formula for a function for $P(t)$ growing at a 3% annual rate with $P(0) = 15$.
2) Write the formula for a function for $P(t)$ growing at a 3% continuous annual rate with $P(0) = 15$.
3) Are your functions in 1 and 2 the same? Write your function in 2 in the form $P(t) = P_0a^t$ for some a and compare with 1.

D) Do problem 38 from Section 1.2 (page 17) of the text. Explain how you are filling in the table.

Assignment

One write-up per group of solutions for these problems, due Thursday, September 8.