*Background*

In class, we have introduced the *left- and right-hand sums* for a function $f$ on an interval $[a, b]$, with a given number of subdivisions $n$, and a step-size $\Delta t = \frac{b-a}{n}$:

$$\text{Left} - \text{hand sum} = f(t_0)\Delta t + \cdots + f(t_{n-1})\Delta t = \sum_{i=0}^{n-1} f(t_i)\Delta t$$

$$\text{Right} - \text{hand sum} = f(t_1)\Delta t + \cdots + f(t_n)\Delta t = \sum_{i=1}^{n} f(t_i)\Delta t$$

We can also do the same sort of thing evaluating $f$ at the midpoint of each smaller interval to get the *midpoint sum*. All of these are examples of *Riemann sums* for $f$.

If $f$ is a continuous function, then the *definite integral* of $f$ is obtained by letting the number $n$ of subdivisions grow without any bound:

$$\int_a^b f(t) \, dt = \lim_{n \to \infty} \sum_{i=0}^{n-1} f(t_i)\Delta t = \lim_{n \to \infty} \sum_{i=1}^{n} f(t_i)\Delta t$$

Today, we want to use some features of Maple to visualize these sums, and the limiting process that yields the value of the definite integral.

*New Maple*

We will be using several commands in the Maple `student` package in this lab. To make these available, you will need to start by entering the command:

```
with(student);
```

to load the student package. The output will be the list of commands included in this package. The ones we will use in the lab are:

- `leftbox, middlebox, rightbox` which draw graphical representations of the left-, midpoint, and right-hand Riemann sums for a given function, and
- `leftsum, middlesum, rightsum` which compute the left-, midpoint, and right-hand Riemann sums of a given function (as formulas). For instance, try entering the following commands to see the pictures for the left- and right-hand sums for $f(x) = t^2 - 3t + 4$ on $[a, b] = [0, 2]$ with $n = 5$ subdivisions:

```
leftbox(t^2 - 3*t + 4, t=0..2,5);
rightbox(t^2 - 3*t + 4, t=0..2,5);
```

To see the numerical values of the left- and right-hand sums, you can enter commands like this:

$$\text{evalf(leftsum(t^2 - 3*t + 4, t=0..2, 5));}$$
$$\text{evalf(rightsum(t^2 - 3*t + 4, t=0..2, 5));}$$

If you leave off the `evalf( )` around the `leftsum` or `rightsum`, can you see what the output means?

As you can probably guess now, the format for all of these commands is: the command name, open paren, the formula for the function $f$, comma, $t =$, then the endpoints, separated by two periods, another comma, then the number $n$, followed by the close paren, then the semicolon.

*Lab Questions*

A) In this question you will consider $f(x) = 4^x$ on the interval $[a, b] = [0, 2]$.

   1) Using the `leftbox` and `rightbox` commands, draw the graphics for the Riemann sums on $[0, 2]$ with $n = 5, 20, 50$. Does it seem reasonable that both left- and right-hand sums will tend to the same limit as $n \to \infty$? Explain in a text region. Also *resize your graphs* before printing out your worksheet. Try to get all these graphs on one page(!)

   2) Using the `leftsum` and `rightsum` commands, compute the numerical values of the sums with $n = 5, 20, 50$. How does these results compare to what you said in part 1?

   3) As we saw in class Wednesday, because $4^x$ is increasing on the interval,

   $$\text{Right} - \text{hand sum} - \text{Left} - \text{hand sum} = (f(2) - f(0))\Delta t = (f(2) - f(0)) \cdot \frac{2 - 0}{n}$$

   and the actual value of the definite integral is somewhere in between the left- and right-hand sum values. How big must $n$ be taken so that the left- and right-hand sums differ by no more than .0001? Using your value $n$, compute the values of the left- and right-hand sums to check your answer.

   4) Now, compute the midpoint sum with $n = 200$. How close is this result to what you had in part 3? What does this say about the numbers of subdivisions needed to approximate $\int_0^2 4^t \, dt$ closely with the different methods?

B) Compute the left-, midpoint, and right-hand sums with $n = 5, 20, 50, 500, 1000$ for each of the following integrals. (No graphics for these, please – just the numerical values.) Observe the limit two which your sums seem to be tending as $n$ increases, and estimate the value of the definite integrals. Also compare the *rates at which* each of the three methods seem to be approaching this limit. Is one getting there faster (i.e. closer to the limit for smaller $n$) than the others?

   1)
   $$\int_0^1 \sin(t^3 - 2t) \, dt$$

2

(In Maple, the function is `sin(t^3 - 2*t)`.)

2)

$$\int_{-1}^{1} e^{-t^2} \, dt$$

(In Maple, the function is `exp(-t^2)`.)

*Assignment*

Individual writeups, due in class Friday, January 25.