Mathematics 134 – Intensive Calculus for Science 2
Things to Know – Final Exam
May 1, 2006

*General Information*

- The final exam will be given in our regular class room 8:30am - 11:30am on Wednesday, May 10.
- This will be a *comprehensive exam* – it will cover all of the material from Chapters 5-11 of the text that we have studied this semester. See the detailed list of topics below.
- The final will be similar in format to the three full period midterm exams, but roughly *twice as long.*
- If you are well prepared and work steadily, it should be possible to complete the exam in about 2 hours. But you will have the full-three hour period to work on the exam if you need that much time.
- *Review Session: Monday, May 8 at 7:00pm.*

*Topics to Be Covered*

1) The definite integral, connections with areas, average values, total change.
2) The Fundamental Theorem of Calculus: 1st part: If $F(x)$ is an antiderivative of a continuous function $f(x)$ on $a \le x \le b$, then

$$\int_a^b f(x)\ dx = F(b) - F(a)$$

   2nd part: If $f(x)$ is continuous for $a \le x \le b$, then the function $F(x) = \int_a^x f(t)\ dt$ is an antiderivative of $f(x)$:

$$\frac{d}{dx}\int_a^x f(t)\ dt = f(x).$$

3) Antiderivatives graphically and numerically
4) The power, sum, and constant multiple rules for antiderivatives
5) Indefinite integrals by substitution, parts, partial fractions, and using the table of integrals
6) Integrals for computing volumes of solids with known cross-sections: $V = \int_a^b A(x)\ dx$, where $A(x)$ is the cross-section area function. Volumes of solids of revolution are a special case of this.
7) Integrals for computing total mass/center of mass of wires and plates of given shapes, given the mass density function
8) Probability density functions of distributions, cumulative distribution function, median, mean.
9) Taylor polynomials and series

10) Taylor series by "shortcut rules" (substitution, algebraic manipulations, term-by-term integration and differentiation) for computing Taylor series from the known series for

$$e^x,\ \sin(x),\ \cos(x),\ (1+x)^p.$$

11) Differential equations
    a) What it means for a function to be a solution of a differential equation
    b) Graphical meaning of solutions via slope fields
    c) Euler's Method for approximate values of solutions
    d) Formulas for solutions via separation of variables
    e) Applications to growth/decay problems

*Suggestions on How to Study*

A) Reread your class notes and work through the examples we did in class – everything on the final will be very close to something we did along the way somewhere!
B) Look over your quizzes and exams, especially any questions you had difficulty with. Read the comments and corrections – they're there to help you figure out what you did wrong and how to solve the problem correctly.
C) Also look at your graded problem sets and the write-ups from the group discussion days and labs.
D) Look over the practice exams for the three full-period midterms.
E) Don't panic! There is a lot of material here, but not all that many key ideas. *Everything on the final will be very close to something we did along the way somewhere!*

*Sample Exam Questions*

The following is essentially the final exam from the last time I taught the Intensive Calculus 2 class. As always, the real exam may look somewhat different, but it will be roughly the same length, of comparable difficulty, and cover most of the same topics.

I. The following table gives the speed (in miles per hour) of a car, measured at 10 minute intervals.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| time (min) | 0 | 10 | 20 | 30 | 40 | 50 | 60 |
| speed (mph) | 35 | 36 | 71 | 23 | 27 | 40 | 35 |

A) Give your best estimate of the total distance traveled based on this information.
B) How often would you need to measure the speed to get the distance accurate to within .1 mile?

II.

A) The following graph shows $y = f(x)$. Find $\int_0^9 f(x)\, dx$ and the average value of $f$ on this interval.

B) Give a precise statement of the Fundamental Theorem of Calculus (both parts).

III. Compute each of the following integrals. If you use an entry from the table, say which one.

A) $\int x^7 - \frac{3}{\sqrt{x}} + \frac{5}{\pi}\, dx$

B) $\int x e^{x^2+1}\, dx$

C) $\int x^2 \cos(3x)\, dx$

D) $\int \frac{1}{\sqrt{1-4x^2}}\, dx$

E) $\int \frac{1}{x^3-9x}\, dx$

IV. Let $R$ be the region bounded by $y = 4 - x^2$, the $x$-axis, and $x = 0$, $x = 1$.

A) Find the volume of the solid obtained by rotating $R$ about the $x$-axis.

B) A thin metal plate has the shape of the region $R$ ($x$ in cm). The mass density of the plate at all points in $R$ is $d(x) = x + 1$ grams per square centimeter. Find the total mass of the plate.

C) What is the $x$-coordinate of the center of mass of the plate from part B? Set up, but *do not evaluate* the integral(s).

V. The amount of snowfall $x$ (in feet) in a remote region of Alaska in the month of January is a random variable with probability density function $p(x) = \frac{2}{9}x(3 - x)$ for $0 \leq x \leq 3$ and 0 otherwise.

A) What is the probability that the amount of snowfall will be between 1 and 2 feet?

B) What is the mean snowfall?

VI.

A) Using the definition of Taylor polynomials, compute the Taylor polynomial of degree $n = 3$ for $f(x) = x\cos(3x)$ at $a = 0$.

B) Check your answer by using our "shortcut methods" to find the first few nonzero terms in the Taylor series for $f(x) = x\cos(3x)$ at $a = 0$.

VII.

A) Show that for any constant $c$, $y = x^2 + \frac{c}{x^2}$ is a solution of the differential equation

$$\frac{dy}{dx} = 4x - \frac{2}{x}y$$

B) Newton's Law of Cooling states that the rate at which the temperature of an object changes is proportional to the difference between the object's temperature and the surrounding temperature. A hot cup of tea with temperature 100°C is placed on a counter in a room maintained at constant temperature 20°C. Ten minutes later the tea has cooled to 76°C. How long will it take to cool off to 45°C? (Express Newton's Law as a differential equation, solve it for the temperature function, then use that to answer the question.)