Mathematics 134 – Intensive Calculus for Science 2
Solutions for Exam 3
April 27, 2006

I.

A) Series A is *not geometric* because there is not a constant ratio between successive terms. For instance $\frac{1/2!}{1} = 1/2$, but $\frac{1/3!}{1/2!} = 2/6 = 1/3$. Series B is geometric with $a = 3/2, r = 1/2$.

B) From the sum formula for infinite geometric series

$$\begin{aligned} 3 &= \frac{a}{1-r} \\ &= \frac{b/2}{1-b/2} \\ \Rightarrow 3 &= \frac{b}{2-b} \\ 6 - 3b &= b \\ 6 &= 4b \\ \Rightarrow b &= 6/4 = 3/2 \end{aligned}$$

II.

A) For $f(x) = \cos(2x)$, by the chain rule:

$$\begin{array}{rl} & f(0) = 1 \\ f'(x) = -2\sin(2x) & f'(0) = 0 \\ f''(x) = -4\cos(2x) & f''(0) = -4 \\ f'''(x) = +8\sin(2x) & f'''(0) = 0 \\ f^{(4)}(x) = 16\cos(2x) & f^{(4)}(0) = 16 \end{array}$$

So the Taylor polynomial is

$$1 + 0 \cdot x + \frac{-4}{2!}x^2 + \frac{0}{3!}x^3 + \frac{16}{4!}x^4 = 1 - 2x^2 + \frac{2}{3}x^4$$

B) Using the general formula for $(1+x)^p$, the first three terms are all nonzero for $p = 1/3$:

$$1 + \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(\frac{-2}{3}\right)}{2}x^2 = 1 + \frac{1}{3}x - \frac{1}{9}x^2.$$

(You would get the same thing by computing the Taylor polynomial of degree 2 using the method of part A too.)

III.

A) It's slope field 2 for the following reason. $\frac{dy}{dx} = (y-1)(y-5)$ has zero slopes (equilibrium solutions) along the lines $y = 1$ and $y = 5$. Slope field 1 doesn't do that (slope is not zero along $y = 1$). There are other ways to tell too, from the signs slopes at various y-values.

B) For slope field 1, the solution with $y(0) = 3$ increases up to a horizontal asymptote at $y = 5$. For slope field 2, the solution with $y(0) = 3$ decreases to a horizontal asymptote at $y = 1$.

C) Separating variables we have

$$\int \frac{dy}{y} = \int \frac{dx}{x(x+1)}$$

The integral on the right can be done with #26 from the table or by the partial fraction method:

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$$

so

$$\ln|y| = \ln|x| - \ln|x+1| + c$$

Exponentiating both sides,

$$y = k\left(\frac{x}{x+1}\right)$$

where $k = \pm e^c$.

IV.

A) The given information says that if Q is the number of bacteria as a function of time, then $\frac{dQ}{dt} = kQ$. This is exponential growth, so

$$Q(t) = Q(0)e^{kt} = 200e^{kt}$$

B) We know

$$360 = Q(.5) = 200e^{(.5)k} \Rightarrow k = \frac{1}{.5}\ln(360/200) \doteq 1.1756$$

So then we want to solve

$$10000 = 200e^{(1.1756)t} \Rightarrow t = \frac{1}{1.1756}\ln(10000/200) \doteq 3.33$$

(hours).