Mathematics 134 – Intensive Calculus for Science 2
Exam 3 – Things To Know
April 21, 2006

General Information

As announced in the course syllabus, the third full-period midterm exam of the semester will be given in class on Friday, April 28. The format will be similar to that of the midterm exams last semester and Exams 1 and 2 this time. It will cover the material since the last exam (Quizzes 7,8,9), including the following material from Chapters 9,10 and 11 of the text:

1) Sequences of real numbers, series (sums), geometric series (including the formulas

$$\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

and

$$\sum_{k=0}^{n} ar^k = \frac{a(1-r^{n+1})}{1-r}$$

(Sections 9.1, 9.2)

2) Taylor polynomials and series (the basic formulas), plus "shortcuts" (substitution, algebraic manipulations, term-by-term integration and differentiation) for computing Taylor series from the known series for

$$e^x,\ \sin(x),\ \cos(x),\ (1+x)^p$$

(Note: You will want to memorize the forms of the Taylor series for these basic examples.) (Sections 10.1, 10.2, 10.3)

3) Differential equations: What it means for a function to be a solution of a differential equation, graphical meaning of solutions via slope fields, approximate solutions via Euler's method, formulas for solutions via separation of variables, growth and decay problems, modeling with differential equations. (Sections 11.1, 11.2, 11.3, 11.4, 11.5, 11.6)

Important Note: These chapters include a number of sections we did not cover in class. As is only fair, *you are not responsible for any of that material.*

A copy of the table of integrals from our text will be provided.

We will review for the exam in class on Wednesday, April 26.

Caution: The following problems indicate *only*

- the kinds of topics that will appear on the exam,
- and the approximate level of difficulty of the questions that will appear.

The actual exam questions *will look somewhat different from these.* The exam itself will be slightly shorter. It is important to understand the methods involved in the solutions to these problems as well as the answers. **Do other practice problems besides these ones and the problems from the problem sets if you want to be well-prepared.**

I. Only one of the two following infinite series is geometric:

$$\text{series A}: 4 - \frac{4}{3} + \frac{4}{9} - \frac{4}{27} + \cdots \qquad \text{series B}: 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots$$

Which one is geometric, and does the geometric series add up to a finite sum? If so, what is the sum? If not, say why not.

II.
A) Compute the 4th degree Taylor polynomial for $f(x) = \ln(1+x)$ at $a = 0$, *using the definition* of Taylor polynomials.
B) Use your polynomial to approximate the value of $\ln(1.1)$. What is the error in your approximation?

III.
A) Using our "shortcut" rules, determine the first three nonzero terms in the Taylor series at $a = 0$ of the functions $\cos(x)$ and $(1-x^2)^{\frac{1}{2}}$.
B) Using the Taylor series, determine which of the two functions in part A is *larger* for small values of x (say $x < .1$). Explain how you can tell.

IV. All parts of this question refer to the differential equation

$$\frac{dy}{dx} = y(4-y)$$

A) Sketch the slope field of this equation, showing the slopes at points on the lines $y = 0, 1, 2, 3, 4, 5$
B) On your slope field, sketch the graph of the solution of the equation with $y(0) = 1$.
C) Also possible questions: problems like those from 11.2 where the plot of the slope field is given, and you need to plot solutions, or identify the formula for the slope function.

V. The population of fish in a lake is attacked by a microscopic water-borne parasite at $t = 0$, and as a result the population declines at a rate proportional to the *square root* of the population from that time on.

A) Express this statement about the rate of growth of the population P as a differential equation.
B) There should be a constant of proportionality, say $-k$, in your equation. Setting $-k = -1$, sketch the slope field for the corresponding differential equation for $t \in [0, 4]$, $P \in [0, 4]$, indicating the slope field segment at each point with integer coordinates.
C) Use $n = 4$ steps of Euler's Method to approximate the value of the solution of your differential equation with $P(0) = 200$ at $t = 1$.
D) Use separation of variables to find the general solution of your differential equation.
E) At $t = 0$ there were 900 fish in the lake; 441 were left after 6 weeks. When did the fish population disappear entirely?

Also see Problem Sets 8,9,10, the last three quizzes, and the Review Problems from the ends of Chapters 9,10,11 in the text for other practice problems.