Mathematics 133 – Intensive Calculus for Science 1
Things to Know – Final Exam
December 5, 2005

General Information

- The final exam will be given in our regular classroom 8:30am - 11:30am on Tuesday, December 13.
- This will be a *comprehensive exam* – it will cover all of the material from Chapters 1 - 5 of the text that we have studied this semester. See the detailed list of topics below.
- The final will be similar in format to the three full period midterm exams, but roughly twice as long.
- If you are well prepared and work steadily, it should be possible to complete the exam in about 2 hours. But you will have the full-three hour period to work on the exam if you need that much time.
- If there is interest, we could try to set up a review session during study week.

Topics to Be Covered

A) The "library of functions" and their properties
   1) Linear functions, the slope and its meaning (linear means constant rate of change), slope-intercept and point-slope forms.
   2) Exponential functions $f(x) = ba^x$, the shapes of the graphs, how to determine a formula from a graph or a table of values, exponential growth vs. exponential decay
   3) Power functions $f(x) = cx^p$.
   4) Inverse functions (when inverse functions exist, how to determine the graph, etc.)
   5) Logarithm functions (Most important: $\log_a(x)$ is the inverse function of a^x, so $y = \log_a(x)$ means the same thing as $x = a^y$.) Natural logarithm is logarithm with base $a = e \doteq 2.71828\cdots$.
   6) Trigonometric functions: $\sin(x), \cos(x), \tan(x)$. Shifted, scaled sinusoidal functions $y = A\sin(Bx) + C$ and $y = A\cos(Bx) + C$.

B) The definition and meaning of the derivative of a function.
   1) The derivative of $f(x)$ at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

   I may ask you to compute a derivative (of a relatively simple function, of course) using this definition
   2) Know how to estimate the derivative of a function given by a table of values numerically
   3) Be able to sketch the 1st and 2nd derivatives of a function given by a graph.

4) $f'(a)$ gives the *instantaneous rate of change of* $f(x)$ at $x = a$. Geometrically, this is the same as the *slope of the tangent line to the graph* $y = f(x)$ at $x = a$.
5) $f'(x) > 0$ on an interval implies f is increasing on that interval; $f'(x) < 0$ implies f is decreasing on that interval.
6) $f''(x) > 0$ on an interval implies that $y = f(x)$ is concave up on that interval; $f''(x) < 0$ on an interval implies that $y = f(x)$ is concave down on that interval.
7) If $f'(c) = 0$ or $f'(c)$ does not exist, we say that c is a critical point. Know how to find critical points of functions given either by formulas or by graphs, and be able to apply the First and Second Derivative Tests for determining the type of the critical point (local maximum, local minimum, or neither).

C) Derivative Rules
1) The power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
2) The exponential rule: $\frac{d}{dx}(a^x) = \ln(a)a^x$ (the derivative of the natural exponential e^x is a special case of this).
3) The rules for trigonometric functions: $\frac{d}{dx}(\sin(x)) = \cos(x)$ and $\frac{d}{dx}(\cos(x)) = -\sin(x)$.
4) The rules for inverse functions: $\frac{d}{dx}(\ln(x)) = \frac{1}{x}$,

$$\frac{d}{dx}(\arcsin(x)) = \frac{1}{\sqrt{1-x^2}}, \qquad \frac{d}{dx}(\arctan(x)) = \frac{1}{1+x^2}$$

5) Sum and constant multiple rules
6) Product rule: $\frac{d}{dx}(f(x)g(x)) = f(x)g'(x) + f'(x)g(x)$
7) Quotient rule:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

8) Chain rule: $\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$.
9) Implicit differentiation

D) Applications of Derivatives
1) First and Second Derivative Tests for local maxima/minima
2) Optimization, or Maximum/Minimum problems
3) Problems about rates of change, including related rates problems.

Suggestions on How to Study

A) Reread your class notes and work through the examples we did in class – everything on the final will be very close to something we did along the way somewhere!
B) Also look at your graded problem sets and the write-ups from the group discussion days.
C) Look over your quizzes and exams, especially any questions you had difficulty with. Read the comments, corrections, and solutions – they're there to help you figure out

what you did wrong and how to solve the problem correctly. *Everything on the final will be very close to something we did along the way somewhere!*

D) Look over the practice exams for the three full-period midterms.

E) Don't panic! There is a lot of material here, but not all that many key ideas.

Practice Final

I.

A) (10) Give a possible formula for the function plotted here:

B) (10) Does the function from part A have an inverse function if x is restricted to the interval $0 \leq x \leq 4$? If so, say why and give the domain of the inverse function. If not, say why not.

II. (15) The function plotted here is a rational function with numerator and denominator both polynomials of degree 2 or less. Find a possible formula.

III. (20) One of the functions tabulated below is approximately linear and the other is approximately exponential. Say which is which and give a formula for either one (your choice).

x	.2	.4	.6	.8	1.0
$f(x)$	2.30	2.51	2.72	2.93	3.14
$g(x)$	2.30	2.42	2.55	2.69	2.84

IV. A cup of hot chocolate is set out on a counter at $t = 0$. The temperature of the chocolate t minutes later is $70 + 80e^{-t/3}$ (in degrees F).

A) (5) What is the temperature of the chocolate at $t = 0$?

B) (10) How fast is the temperature changing at $t = 10$ (give units).

C) (10) How long does it take for the temperature to reach 100°F?

V.

A) (10) Using the limit definition, compute $f'(x)$ for $f(x) = \frac{1}{x+1}$.

Using appropriate derivative rules, compute the derivatives of the following functions:

B) (10) $g(x) = 3x^4 + \frac{3}{\sqrt{x}} + 2\sqrt[3]{x} + \pi^2$.

C) (10) $g(x) = \frac{\tan(x)+x}{\cos(2x)}$

D) (10) $i(x) = 3\ln(x^2 + 3^x)$

E) (10) $j(x) = \arcsin(12x + 2)$

VI. (10) The following graph shows $y = f(x)$. Make a qualitative sketch of the graph $y = f'(x)$.

VII. The following graph shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $-3/2 \le x \le 3/2$. Using the graph, *estimate*

A) (5) The intervals on which f is increasing.

B) (10) The critical points of $f(x)$. Say what the type (local max, local min, neither) of each critical point is.

C) (7.5) The intervals on which $y = f(x)$ is concave up.

D) (7.5) The locations of the inflection points of $y = f(x)$.

VIII. (20) A window is to be made of a single large piece of glass in the shape of a semicircle on top of a rectangle (the diameter of the semicircle is the same as the width of the rectangle). The perimeter of the window is 60 ft. What dimensions will maximize the total area of the window?

IX. (10) Compute the LHS and RHS approximations to $\int_1^4 x^2 + x - 3\ dx$ using a partition with $n = 6$ subdivisions.