I. First note that \( f \) cannot be either linear or exponential because it is increasing for some \( x \) and decreasing for other \( x \).

A) \( g(x) \) is the only linear function, because it is the only one with constant slope. Using the first two points for instance \( m = \frac{28-2}{-2} = \frac{5}{2} \). The \( y \)-intercept is 28 from the table so \( g(x) = \frac{5}{2}x + 28 \).

B) \( h(x) \) is exponential. This can be seen because the ratios of successive values is constant:

\[
\frac{18}{2} = \frac{2}{2/9} = \frac{2/9}{2/81} = \frac{2/81}{2/729} = 9.
\]

To find the formula, we try to determine \( a, b \) to match the table with \( h(x) = ba^x \).
From \( h(0) = 2 \), we get \( 2 = ba^0 = b \). Then from \( h(-2) = 18 \) we get \( 18 = 2a^{-2} \) so \( 9 = a^{-2} \), so \( a^2 = 1/9 \), and \( a = 1/3 \). The formula is \( h(x) = 2 \left( \frac{1}{3} \right)^x \).

II. Because the “Explosium” decays exponentially, we know the amount is given by a formula \( Q(t) = Q_0e^{kt} \) or \( Q(t) = Q_0a^t \). Either form can be used, and we will show how to do the problem using both of them.

*Method 1.* Using \( Q(t) = Q_0e^{kt} \), the initial amount is \( Q_0 = 10 \), and the half-life is 12 years so

\[
\frac{5}{10} = e^{k \cdot 12},
\]

\[
\frac{1}{2} = e^{12k},
\]

\[
\ln(1/2) = 12k,
\]

\[
\frac{\ln(1/2)}{12} = k.
\]

Then we want to determine the time \( t \) when \( Q(t) = 1 \):

\[
1 = 10e^{\frac{\ln(1/2)}{12}t},
\]

\[
1/10 = e^{\frac{\ln(1/2)}{12}t},
\]

\[
\ln(1/10) = \frac{\ln(1/2)}{12}t,
\]

\[
\frac{12\ln(1/10)}{\ln(1/2)} = t.
\]

*Method 2.* Using \( Q(t) = Q_0a^t \), we have \( Q = 10a^t \) and substituting \( t = 12 \) (the half-life) we get \( 5 = 10a^{12} \), so \( a = \left( \frac{1}{2} \right)^{1/12} \) and \( Q = 10 \left( \frac{1}{2} \right)^{1/12} \). Then as before we want
to solve for $t$ in the equation $1 = 10 \left( \frac{1}{2} \right)^{t/12}$. We get:

$$1/10 = \left( \frac{1}{2} \right)^{t/12}$$

$$\ln(1/10) = (t/12)\ln(1/2)$$

$$12\ln(1/10) = t$$

(which agrees with the other method, of course!)

III. A) Because this plot is sinusoidal, with a maximum at $x = 0$, it is simplest to use
the form $y = A \cos(Bx) + C$. The amplitude $A = (5 - (-3))/2 = 4$. The period is 1, so
$2\pi/B = 1$ and $B = 2\pi$. The graph is also shifted vertically by $C = -1$:

$$y = 4 \cos(2\pi x) - 1.$$  

B) This graph looks like a polynomial of degree 3 since there are 2 turning points, a double
root at $x = 1$, and a root at $x = 2$. This gives $y = k(x - 1)^2(x - 2)$ for some constant
$k$. Since the graph “starts high and finishes low” we know $k < 0$. The exact value can be
determined because the graph shows the $y$-intercept at $y = 6$:

$$6 = k(0 - 1)^2(0 - 2) = -2k$$

So $k = -3$ and the formula is

$$y = -3(x - 1)^2(x - 2).$$

IV. The graph $y = 10^{x+1}$ is an exponential growth curve (shifted left by 1). So it does
pass the horizontal line test (horizontal lines $y = c > 0$ intersect the graph exactly once).
This says there is an inverse function. To find the formula, interchange $x$ and $y$ then solve
for $y$ taking logarithms:

$$x = 10^{y+1}$$

$$\ln(x) = (y + 1)\ln(10)$$

$$\frac{\ln(x)}{\ln(10)} = y + 1$$

$$\frac{\ln(x)}{\ln(10)} - 1 = y$$

The inverse function is $f^{-1}(x) = \frac{\ln(x)}{\ln(10)} - 1$.

V. A) $r(x)$ has vertical asymptotes where the bottom is zero: $16 - x^2 = 0$, so $x = \pm 4$.
B) $r(x)$ has a horizontal asymptote at $y = -2$ because as $x \to \pm\infty$,

$$\frac{2x^2 - x}{16 - x^2} = \frac{2 - \frac{1}{x}}{16x^2 - 1} \to -2$$

C) The graph $y = r(x)$ crosses the $x$-axis where the top is zero: $0 = 2x^2 - x = x(2x - 1)$,
so $x = 0$ or $x = 1/2$. 

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