```
Id := [x+y+z+w, x·y+y·z+z·w+w·x, x·y·z+y·z·w+z·w·x+w·x·y, x·y·z·w-1];
```

$$[x+y+z+w,\ wx+wz+xy+yz,\ wxy+wxz+wyz+xyz,\ wxyz-1] \quad (1)$$

```
BC4 := Groebner[Basis](Id, tdeg(x, y, z, w));
```

$$[x+y+z+w,\ w^2+2wy+y^2,\ -w^3-w^2y+wz^2+yz^2,\ -w^4-w^3y+w^3z+w^2yz + w^2z^2-1,\ w^5+w^4y-w-y,\ w^3z^2+w^2z^3-w-z,\ w^4z^2-2w^2-wy+wz + yz] \quad (2)$$

```
> nops(BC4);
```

$$7 \quad (3)$$

```
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
&
&  CALCULATION REDONE WITH CORRECT ComputeM
&
&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&
>
>
>
>
coefflist :=proc(poly, monlist, termorder)
  local lm, lc, rest, list, i;
  rest := poly;
  list := [seq(0, i = 1..nops(monlist))]:
  while rest ≠ 0 do
     lm := LeadingMonomial(rest, termorder);
     lc := LeadingCoefficient(rest, termorder);
#     lprint(lc, lm);
    i := 1;
    while lm ≠ monlist[i] do
      i := i+1;
#       lprint(i);
    end do;
    list[i] := lc;
    rest := expand(rest-lc·lm);
 #    lprint(rest);
  end do:
  return(list);
  end proc:

> with(Groebner):
> ComputeM :=proc(L, G, torder)

     local H, ddone, mon, mat, AllTerms, AllMonoms, AllMonomsList, SortedMonoms,
       j, xbeta, k, M;

     H := convert(L, set);
     ddone := {seq(LeadingMonomial(H[i], torder), i = 1..nops(H))};
```

```
  AllTerms := {seq(op(H[i]), i = 1 ..nops(H) )};
#   lprint(ddone, H, AllTerms);
  AllMonoms := {seq(LeadingMonomial(AllTerms[j], torder), j = 1
    ..nops(AllTerms) )};
  AllMonomsList := convert(AllMonoms, list);
  SortedMonoms := sort(AllMonomsList, (a, b) → TestOrder(b, a, torder) );
    #Mon(H) as a list
#   print(SortedMonoms);
  while ddone ≠ convert(SortedMonoms, set) do
     j := 1;
    while evalb(SortedMonoms[j] in ddone) do
      j := j + 1;
    end do;
    if j ≤ nops(SortedMonoms) then
       xbeta := SortedMonoms[j];
#         print(xbeta);
       ddone := ddone union {xbeta};
       k := 1;
      while (k ≤ nops(G)) and denom( xbeta / LeadingMonomial(G[k], torder) ) ≠ 1 do
         k := k + 1;
      end do;
      if k ≤ nops(G) then
        H := H union {expand( xbeta / LeadingMonomial(G[k], torder) · G[k] )};
      end if;
    end if;
    AllTerms := {seq(op(H[i]), i = 1 ..nops(H) )};
    AllMonoms := {seq(LeadingMonomial(AllTerms[j], torder), j = 1
    ..nops(AllTerms) )};
    AllMonomsList := convert(AllMonoms, list);
    SortedMonoms := sort(AllMonomsList, (a, b) → TestOrder(b, a, torder) );
    #Mon(H) as a list
  end do;
#  print(H);
  M := Matrix( [seq(coefflist(H[j], SortedMonoms, torder), j = 1 ..nops(H) )] );
  return(M, SortedMonoms)
  end proc:
```

```
> Mat2 := ComputeM( [expand(y·Id[1]), Id[2]], Id, tdeg(x, y, z, w) )[1];
```

$$Mat2 := \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{4}$$

```
> MonList2 := ComputeM([expand(y·Id[1]), Id[2]], Id, tdeg(x, y, z, w))[2];
```

$$MonList2 := [yx, y^2, yz, wx, yw, wz, w^2] \tag{5}$$

```
> with(LinearAlgebra):
> interface(rtablesize = infinity):
```

```
> ReducedRowEchelonForm(Mat2);
```

$$\begin{bmatrix} 1 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{6}$$

```
> Id := [op(Id), y^2 + 2·y·w + w^2];
```

$$Id := [x+y+z+w,\ wx+wz+xy+yz,\ wxy+wxz+wyz+xyz,\ wxyz-1,\ w^2+2wy+y^2] \tag{7}$$

```
> Mat3 := ComputeM(map(x→expand(x), [y·z·Id[1], Id[3], y^2·Id[1], x·Id[5], z
      ·Id[2], Id[3], y·Id[2], x·Id[5]]), Id, tdeg(x, y, z, w))[1];
```

$$Mat3 := \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{8}$$

```
> MonList3 := ComputeM(map(x→expand(x), [y·z·Id[1], Id[3], y^2·Id[1], x·Id[5], z
      ·Id[2], Id[3], y·Id[2], x·Id[5]]), Id, tdeg(x, y, z, w))[2];
```

$$MonList3 := [y^2x, y^3, xyz, y^2z, yz^2, wxy, y^2w, wxz, wyz, wz^2, w^2x, w^2y, w^2z, w^3] \tag{9}$$

```
>
> RM3 := ReducedRowEchelonForm(Mat3);
```

$$RM3 := \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & -2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \textbf{(10)}$$

pairs: {1,3},{1,4},{1,5},{2,3},{2,4},{2,5},{3,4},{3,5},{4,5}
degrees 3 4 3 3 4 3 4 4 5

B' = {1,3}, {1,5}, {2,3}, {2,5}

```
L = {yz*f1 = x*y*z + y^2*z + y*z^2 + y*z*w,
     f3 = x*y*z +y*z*w+z*w*x+w*x*y,
     y^2*f1 = x*y^2 + y^3 + z*y^2 + w*y^2,
     x*f5 = x*y^2 - x^2*w + x*y*w - x*z*w,
     z*f2 = x*y*z + y*z^2 + z^2*w + z*x*w,
     f3 = x*y*z + y*z*w+z*w*x+w*x*y,
     y*f2 = x*y^2 + y^2*z+y*z*w+x*y*w,
     x*f5 = x*y^2 + 2*x*y*w + x*w^2}
```

```
> for i to RowDimension(RM3) do
    poly := 0 :
    for j to ColumnDimension(RM3) do
      poly := poly + RM3[i, j]·MonList3[j];
    end do:
    lprint(sort([op(poly)], (a, b) → TestOrder(b, a, tdeg(x, y, z, w))));
  end do:
[y^2*x, -2*w*y*z, w^2*y, -w^2*z, w^3]
[y^3, -3*w^2*y, -2*w^3]
[x*y*z, -w*y*z, -w*z^2, w^2*y, -w^2*z, w^3]
[y^2*z, 2*w*y*z, w^2*z]
[y*z^2, w*z^2, -w^2*y, -w^3]
[w*x*y, w*y*z, -w^2*y, -w^3]
[y^2*w, 2*w^2*y, w^3]
[w*x*z, w*y*z, w*z^2, w^2*z]
[w^2*x, w^2*y, w^2*z, w^3]
```

[0]
[0]
[0]

> $f6 := add(RM3[5, j] \cdot MonList3[j], j = 1..nops(MonList3))$;

$$f6 := -w^3 - w^2 y + w z^2 + y z^2 \quad (11)$$

> $BC4[3]$;

$$-w^3 - w^2 y + w z^2 + y z^2 \quad (12)$$

> $Id := [op(Id), f6]$;

$$Id := [x + y + z + w,\ w x + w z + x y + y z,\ w x y + w x z + w y z + x y z,\ w x y z - 1,\ w^2 + 2 w y + y^2,\ -w^3 - w^2 y + w z^2 + y z^2] \quad (13)$$

> $Mat4 := ComputeM(map(x \to expand(x), [y \cdot z \cdot w \cdot Id[1], Id[4], z \cdot w \cdot Id[2], w \cdot Id[3], y \cdot Id[3], x \cdot z \cdot Id[5], y \cdot z^2 \cdot Id[1], x \cdot Id[6], z^2 \cdot Id[2], z \cdot Id[3], y \cdot Id[6], z^2 \cdot Id[5]]), Id, tdeg(x, y, z, w))[1]$;

$$Mat4 := \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0
\end{bmatrix} \quad (14)$$

> $RM4 := ReducedRowEchelonForm(Mat4);$

$$RM4 := \left[\begin{array}{cccccccccccccccccccccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 2 & 0 & 2 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & -1 & 1 & -1 & -2 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -3 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 & 2 & -1 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & -1 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right] \quad \textbf{(15)}$$

> $MonList4 := ComputeM\big(map\big(x \to expand(x), [y \cdot z \cdot w \cdot Id[1], Id[4], z \cdot w \cdot Id[2], w \cdot Id[3], y \cdot Id[3], x \cdot z \cdot Id[5], y \cdot z^2 \cdot Id[1], x \cdot Id[6], z^2 \cdot Id[2], z \cdot Id[3], y \cdot Id[6], z^2 \cdot Id[5]]\big), Id, tdeg(x, y, z, w)\big)[2];$

$$MonList4 := [x\,y^2\,z,\ y\,z^2\,x,\ y^2\,z^2,\ y\,z^3,\ w\,x\,y^2,\ y^3\,w,\ w\,x\,y\,z,\ w\,y^2\,z,\ w\,x\,z^2,\ w\,y\,z^2,\ w\,z^3,\ w^2\,x\,y,\ y^2\,w^2,\ w^2\,x\,z,\ w^2\,y\,z,\ w^2\,z^2,\ w^3\,x,\ w^3\,y,\ w^3\,z,\ w^4,\ 1] \quad \textbf{(16)}$$

```
> for i to RowDimension(RM4) do
    poly := 0 :
    for j to ColumnDimension(RM4) do
      poly := poly + RM4[i, j]·MonList4[j];
    end do:
    lprint(sort([op(poly)], (a, b) → TestOrder(b, a, tdeg(x, y, z, w))));
  end do:
```

```
[x*y^2*z, -w^3*y, -w^4, 1]
[y*z^2*x, -w*z^3, -w^2*z^2, 1]
[y^2*z^2, -w^2*z^2, 2*w^3*y, 2*w^4]
[y*z^3, w*z^3, w^2*z^2, -w^3*y, -w^4, -1]
[w*x*y^2, 2*w^2*z^2, -w^3*y, w^3*z, -w^4, -2]
[y^3*w, -3*w^3*y, -2*w^4]
[w*x*y*z, -1]
[w*y^2*z, -2*w^2*z^2, 2*w^3*y, -w^3*z, 2*w^4, 2]
[w*x*z^2, w*z^3, w^3*y, w^4]
[w*y*z^2, w^2*z^2, -w^3*y, -w^4]
[w^2*x*y, -w^2*z^2, -w^3*z, 1]
[y^2*w^2, 2*w^3*y, w^4]
[w^2*x*z, w^3*y, w^4, 1]
[w^2*y*z, w^2*z^2, -w^3*y, w^3*z, -w^4, -1]
[w^3*x, w^3*y, w^3*z, w^4]
[0]
[0]
[0]
[0]
[0]
[0]
[0]
```

> $f7 := add(RM4[14, j] \cdot MonList4[j], j = 1..nops(MonList4))$;

$$f7 := -w^4 - w^3 y + w^3 z + w^2 y z + w^2 z^2 - 1 \quad (17)$$

> $BC4[4]$;

$$-w^4 - w^3 y + w^3 z + w^2 y z + w^2 z^2 - 1 \quad (18)$$

> $Id := [op(Id), f7]$;

$$Id := [x + y + z + w,\ w x + w z + x y + y z,\ w x y + w x z + w y z + x y z,\ w x y z - 1,\ w^2 + 2 w y + y^2,\ -w^3 - w^2 y + w z^2 + y z^2,\ -w^4 - w^3 y + w^3 z + w^2 y z + w^2 z^2 - 1] \quad (19)$$

> $Mat5 := ComputeM(map(x \to expand(x), [w^2 \cdot y \cdot z \cdot Id[1], x \cdot Id[7], w^2 \cdot z \cdot Id[2], w^2 \cdot Id[3], y \cdot Id[4], x \cdot z \cdot w \cdot Id[5], z \cdot Id[4], x \cdot w \cdot Id[4], w \cdot Id[4], x \cdot Id[7], w^2 \cdot z \cdot Id[5], y \cdot Id[7], w^2 \cdot Id[6], z \cdot Id[7]]), Id, tdeg(x, y, z, w))[1]$;

$$Mat5 := [[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1],$$
$$[0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0],$$
$$[0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0],$$
$$[0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1],$$
$$[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,0,0,0,1,0,0,0,0,2,0,1,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,1,0,0,0,0,0,0,2,0,0,0,1,0,0,0,0,0,0,0,0,0],$$
$$[0,1,0,2,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,1,0,0,0,0,0,1,0,-1,0,-1,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0], \quad (20)$$

[0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, -1, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, -1, 0],
[0, 0, 0, 0, 1, 0, 1, 0, 0, -1, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, -1, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, -1, 1, -1, 0, 0, 0, 0, 0, 0, -1],
[0, 0, 0, 1, 0, 1, 0, 0, -1, 0, 1, 0, 0, -1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0]]

> $MonList5 := ComputeM\left(map\left(x \to expand(x), \left[w^2 \cdot y \cdot z \cdot Id[1], x \cdot Id[7], w^2 \cdot z \cdot Id[2], w^2 \cdot Id[3], y \cdot Id[4], x \cdot z \cdot w \cdot Id[5], z \cdot Id[4], x \cdot w \cdot Id[4], w \cdot Id[4], x \cdot Id[7], w^2 \cdot z \cdot Id[5], y \cdot Id[7], w^2 \cdot Id[6], z \cdot Id[7]\right]\right), Id, tdeg(x, y, z, w)\right)[2];$

$MonList5 := [w^2 x^2 y z, w x y^2 z, w x y z^2, w^2 y z x, w^2 y^2 z, w^2 x z^2, w^2 y z^2, w^2 z^3, w^3 x y, w^3 y^2, w^3 x z, w^3 y z, w^3 z^2, w^4 x, w^4 y, w^4 z, w^5, w x, y w, w z, w^2, x, y, z, w]$ **(21)**

> $RM5 := ReducedRowEchelonForm(Mat5);$

RM5 := [[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 0, 0], **(22)**
[0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, 0],
[0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0],
[0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1],
[0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, -2, 0, 0, -1, 0, 0, 0, 0, 0, 0, 2, 0, 4],
[0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2],
[0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, -1, 0, -1],
[0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, -1, -1],
[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, -1, 0, 0, -1, 0, 0, 0, 0, 0, 0, 0, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, -1, 0, 0, 0, 0, 0, 2, 0, 2],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, -1, 0, -2],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, -1, 0, -1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],

```
    [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]
```

```
> for i to RowDimension(RM5) do
    poly := 0 :
    for j to ColumnDimension(RM5) do
      poly := poly + RM5[i, j]·MonList5[j];
    end do:
    lprint(sort([op(poly)], (a, b) → TestOrder(b, a, tdeg(x, y, z, w))));
  end do:
```

```
[w^2*x^2*y*z, y*w, w*z, w^2]
[w*x*y^2*z, -y]
[w*x*y*z^2, -z]
[w^2*y*z*x, -w]
[w^2*y^2*z, -2*w^3*z^2, -w^4*z, 2*y, 4*w]
[w^2*x*z^2, -w^3*z^2, y, z, 2*w]
[w^2*y*z^2, w^3*z^2, -y, -w]
[w^2*z^3, w^3*z^2, -z, -w]
[w^3*x*y, -w^3*z^2, -w^4*z, w]
[w^3*y^2, -w^5, 2*y, 2*w]
[w^3*x*z, y, 2*w]
[w^3*y*z, w^3*z^2, w^4*z, -y, -2*w]
[w^4*x, w^4*z, y, w]
[w^4*y, w^5, -y, -w]
[w*x, y*w, w*z, w^2]
[x, y, z, w]
[0]
[0]
[0]
[0]
[0]
```

```
> f8 := add(RM5[8, j]·MonList5[j], j = 1..nops(MonList5));
```

$$f8 := w^3 z^2 + w^2 z^3 - w - z \tag{23}$$

```
> f9 := add(RM5[14, j]·MonList5[j], j = 1..nops(MonList5));
```

$$f9 := w^5 + w^4 y - w - y \tag{24}$$

```
> Id := [op(Id), f8, f9];
```

$$Id := [x + y + z + w,\ w x + w z + x y + y z,\ w x y + w x z + w y z + x y z,\ w x y z - 1,\ w^2 + 2 w y + y^2,\ -w^3 - w^2 y + w z^2 + y z^2,\ -w^4 - w^3 y + w^3 z + w^2 y z + w^2 z^2 - 1,\ w^3 z^2 + w^2 z^3 - w - z,\ w^5 + w^4 y - w - y] \tag{25}$$

```
> BC4[5];
```

$$w^5 + w^4 y - w - y \tag{26}$$

```
> BC4[6];
```

$$w^3 z^2 + w^2 z^3 - w - z \tag{27}$$

```
> Mat6 := ComputeM(map(x → expand(x), [w^2·z^3·Id[1], x·Id[8], w^4·y·Id[1], x·Id[9],
      w^4·Id[2], w^4·Id[5], y·Id[9], w^2·z·Id[6], y·Id[8], z^2·Id[7], w^2·Id[7], z·Id[9]]),
```

```
    Id, tdeg(x, y, z, w))[1];
```

$$Mat6 := [[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,2,0,1],$$
$$[0,0,0,0,0,0,0,1,0,0,0,2,0,1,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,-1,0,0,0,0,-1,0,0],$$
$$[0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,0,0,0,-1,0,-1],$$
$$[0,0,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,-1,0],$$
$$[0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,-1,-1],$$
$$[0,0,0,0,0,0,1,0,0,0,1,0,0,0,-1,0,0,0,0,-1,0,0,0],$$
$$[1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,-1,0,0,0],$$
$$[0,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,-1,0,0],$$
$$[0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,-1,0,0,0,-1,0],$$
$$[0,1,0,0,0,1,0,0,-1,0,0,0,-1,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,1,0,0,0,0,1,0,-1,0,-1,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,0,0,1,0],$$
$$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,1,0,0],$$
$$[0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,1,1],$$
$$[1,1,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,1,1,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,1,1,1,0,0,1,0,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,1,0,1,0,1,0,1,0,0,0,0,0,0,0,0,0,0],$$
$$[0,0,0,0,0,0,0,0,0,0,1,1,1,1,0,0,0,0,0,0,0,0,0],$$
$$[0,1,1,0,-1,1,0,0,0,-1,0,0,0,0,0,0,0,-1,0,0,0,0],$$
$$[0,0,0,0,0,0,0,0,1,1,0,-1,1,-1,0,0,0,0,0,0,0,0,-1]]$$

```
> MonList6 := ComputeM(map(x→expand(x), [w^2·z^3·Id[1], x·Id[8], w^4·y·Id[1], x
    ·Id[9], w^4·Id[2], w^4·Id[5], y·Id[9], w^2·z·Id[6], y·Id[8], z^2·Id[7], w^2·Id[7], z
    ·Id[9]]), Id, tdeg(x, y, z, w))[2];
```

$$MonList6 := [w^2 z^3 x,\ w^2 z^3 y,\ w^2 z^4,\ w^3 x z^2,\ w^3 y z^2,\ w^3 z^3,\ w^4 y x,\ w^4 y^2,\ w^4 y z,$$
$$w^4 z^2,\ w^5 x,\ w^5 y,\ w^5 z,\ w^6,\ y x,\ y^2,\ x z,\ y z,\ z^2,\ w x,\ y w,\ w z,\ w^2]$$

```
> RM6 := ReducedRowEchelonForm(Mat6);
```

$$RM6 := [[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,1,0,1],$$
$$[0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,-1,1,-1],$$
$$[0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,-1,0,-1,1,-1],$$
$$[0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,2,0],$$
$$[0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,0,-1,1],$$
$$[0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,-1,0,0,1,-2,1],$$
$$[0,0,0,0,0,0,1,0,0,0,0,0,0,-1,0,0,0,0,1,0,0,-1,1,-1],$$
$$[0,0,0,0,0,0,0,1,0,0,0,0,0,-1,0,0,0,0,0,0,2,0,2],$$
$$[0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,-1,0,0,0,-1,0],$$

[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, -1, 1, -2],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, -1, 0, -1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, -1, 0, -1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 0, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 0, 0, 1, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0],
[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]]

```
> for i to RowDimension(RM6) do
    poly := 0 :
    for j to ColumnDimension(RM6) do
      poly := poly + RM6[i, j]·MonList6[j];
    end do:
    lprint(sort([op(poly)], (a, b) → TestOrder(b, a, tdeg(x, y, z, w))));
  end do:
```

```
[w^2*z^3*x, z^2, y*w, w^2]
[w^2*z^3*y, -y*w, w*z, -w^2]
[w^2*z^4, y*z, -z^2, -y*w, w*z, -w^2]
[w^3*x*z^2, y*z, 2*w*z]
[w^3*y*z^2, -y*z, -w*z, w^2]
[w^3*z^3, -y*z, y*w, -2*w*z, w^2]
[w^4*y*x, -w^5*z, y*z, -y*w, w*z, -w^2]
[w^4*y^2, -w^6, 2*y*w, 2*w^2]
[w^4*y*z, w^5*z, -y*z, -w*z]
[w^4*z^2, y*z, -y*w, w*z, -2*w^2]
[w^5*x, w^5*z, y*w, w^2]
[w^5*y, w^6, -y*w, -w^2]
[y*x, y*z, -y*w, -w^2]
[y^2, 2*y*w, w^2]
[x*z, y*z, z^2, w*z]
[w*x, y*w, w*z, w^2]
[0]
[0]
[0]
[0]
[0]
[0]
```

```
> BC4[7];
```

$$w^4 z^2 - 2 w^2 - w y + w z + y z \qquad (31)$$

```
> F := add(RM6[10, j]·MonList6[j], j = 1..nops(MonList6));
```

$$F := w^4 z^2 - 2 w^2 - w y + w z + y z \tag{32}$$