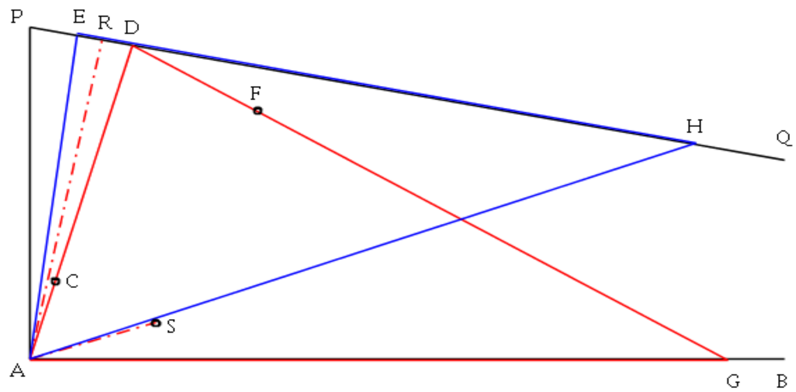

Proposition 2 *If $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{AB}$ in the direction of B (according to Gauss's definition of parallelism), then $\overleftrightarrow{AB}$ is also parallel to $\overleftrightarrow{PQ}$ in the direction of Q, provided B, Q are on the same side of the segment AP.*

The construction:

- Drop $AR \perp PQ$
- Say S is in $ABQR \subset ABQP$. The goal is to show that $\overleftrightarrow{AS}$ meets $\overleftrightarrow{PQ}$. Call $\angle SAB = \alpha$.
- Choose C so that $\angle RAC = \frac{1}{2}\alpha$
- Assume $\overrightarrow{AC}$ meets $\overleftrightarrow{PQ}$ at D.
- Let $RE = RD$, with E on the other side of D. Hence $\angle SAB = \angle EAD = \alpha$ and $AE = AD$. (Why?)
- Choose F so $\angle ADF = \angle AED$. Since $\overleftrightarrow{PQ}$ is the parallel to $\overleftrightarrow{AB}$ in the direction of B, the line $\overleftrightarrow{DF}$ meets the line $\overleftrightarrow{AB}$ at some point G (see Proposition 1 from Monday).
- Let H be on $\overleftrightarrow{PQ}$ with $EH = DG$ and join AH.

Claim. ΔAEH (outlined in blue) $\cong \Delta ADG$ (outlined in red). (Why?)

Conclusion. $\angle GAH = \angle GAS$, so S is on the line $\overleftrightarrow{AH}$, which meets $\overleftrightarrow{PQ}$, which is what we wanted to show. (Why?)
What if $\overrightarrow{AC}$ does not meet $\overleftrightarrow{PQ}$?