MATH 392 – Geometry Through History
Problem Set 3
Due: Monday, February 29

I. Show under the assumption of HAA that given any angle $\angle ABC < 180°$, there is a line ℓ that is simultaneously parallel to $\overrightarrow{BA}$ in the direction of A and to $\overrightarrow{BC}$ in the direction of C. Hint: use point (3) on page 47 of McCleary.

II. Note: All of the following facts about 3-dimensional geometry are proved in McCleary. Suggestion: try to work out proofs for yourselves using just properties following from Euclid's Postulates I-IV (not V), then use the text to check your work.

(A) Show that the following are equivalent for a line ℓ intersecting a plane T at a point P.

   (1) $\ell \perp T$ (i.e. ℓ meets every line through P in T at a right angle – see McCleary and the handout for Wednesday and Friday).

   (2) There exist lines $m_1 \neq m_2$ through P in T such that ℓ meets m_1 and m_2 at right angles.

(B) If $\ell \perp T$ with $\ell \cap T = P$, let m be any line in T and $A \in m$ such that $\overline{AP} \perp m$. If B is any point on ℓ, then $\overline{AB} \perp m$.

(C) If T and T' are perpendicular planes, and ℓ is a line on T, then $\ell \perp T'$ is equivalent to $\ell \perp m$ for $m = T \cap T'$.

(D) If $\ell_1 \perp T$ and $\ell_2 \perp T$, then ℓ_1, ℓ_2 are coplanar and non-intersecting (but not necessarily parallel – this is a difference in the hyperbolic case!)

(E) Given a line ℓ and a plane T not containing ℓ, there exists a unique plane T' containing ℓ that is perpendicular to T.

III. Let $\cosh(t) = \frac{e^t + e^{-t}}{2}$ and $\sinh(t) = \frac{e^t - e^{-t}}{2}$ be the hyperbolic functions.

(A) Sketch the graphs $y = \cosh(t)$ and $y = \sinh(t)$. (Suggestion: plot $y = e^t$ and $y = e^{-t}$ first, then combine them in appropriate ways.)

(B) Show that $\lim_{t \to \pm\infty} \tanh(t) = 1$ and sketch the graph $y = \tanh(t)$.

(C) Determine the Taylor series of the functions $\cosh(t)$ and $\sinh(t)$ at $t = 0$.

(D) Show that $\cosh^2(t) - \sinh^2(t) = 1$ for all real t. Explain why this shows that the parametric curve $x = \cosh(t), y = \sinh(t)$ lies on the hyperbola with equation $x^2 - y^2 = 1$. What part of that hyperbola is obtained from the parametrization?

IV. Why is the formula

$$\cosh(c/k) = \cosh(a/k)\cosh(b/k)$$

called the hyperbolic form of the Pythagorean theorem? Hint: Use III, part (C) and think about what happens as $k \to \infty$.