MATH 392 – Geometry Through History
Problem Set 2
Due: Friday, February 12

I. Proposition 30 in Book I of Euclid's *Elements* says: *Two lines that are parallel to the same line are parallel to each other.*

Here's the proof Euclid provides: Let it be given that the lines AB and and EF are parallel and the lines CD and EF are parallel. Pick any two points G on the line AB and K on the line CD, and draw the line segment GK (and extend it – Postulates I and II). Let H be the intersection of the line GK with the line EF. Then $\angle AGH = \angle FHG$ and $\angle FHG = \angle DKG$ (Proposition 29). Hence also $\angle AGH = \angle DKG$ (Common Notion 1). Therefore, AB is parallel to CD (Proposition 27).

(A) There is a point here that is questionable in the sense that it does not really follow from any of the Postulates or the previous results in Book I. What is this issue?

(B) To fix the problem you noted in part (A), prove the following statement: *If a line intersects one of a pair of parallel lines, then it intersects the other one as well.* Hint: You should take "line" here to mean a line extended as far as possible in both directions (Postulate 2). Argue by contradiction and note that two lines that do not intersect are parallel by definition. Your proof *should* make use of results that depend on Postulate V.

II. Give a proof of Proposition 35 in Book I of the *Elements* in the cases that the sides CD and EF of the two parallelograms overlap. (See the slides on Book I.)

III. Assume Postulates I - IV of the *Elements* hold. Consider the following statements:

(A) Euclid's Postulate V.

(B) A line perpendicular to one ray of an acute angle intersects the other ray as well.

(C) Through any point in the interior of an angle less than a straight angle (180°), there passes a line meeting each of the two rays at points other than the vertex.

(D) The sum of the angles in any triangle is 180°.

Show that all these statements are equivalent by showing that (A) $\Rightarrow$ (B), (B) $\Rightarrow$ (C), (C) $\Rightarrow$ (D). Hints: We know from class that (D) $\Rightarrow$ (A). So once you show those three implications, all four statements are equivalent. Also, see Problem 3.5 in McCleary, which gives an "attempted proof" of Postulate V by A.-M. Legendre (1804). You *can* use this proof under the assumption that (C) is true. Do you see why? And do you see why Legendre's proof fails if we don't know (C) is true?