MATH 392 – Geometry Through History
Syllabus – Spring 2016

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> Euclid alone has looked on Beauty bare.
> Let all who prate of Beauty hold their peace,
> And lay them prone upon the earth and cease
> To ponder on themselves, the while they stare
> At nothing, intricately drawn nowhere
> In shapes of shifting lineage; let geese
> Gabble and hiss, but heroes seek release
> From dusty bondage into luminous air.
>
> O blinding hour, O holy, terrible day,
> When first the shaft into his vision shone
> Of light anatomized! Euclid alone
> Has looked on Beauty bare. Fortunate they
> Who, though once only and then but far away,
> Have heard her massive sandal set on stone.
>
> *Edna St. Vincent Millay, 1892-1950*

Course Description

Geometry has been a central part of mathematics since the time of the ancient Greeks. The understanding of plane and solid geometry achieved and codified in the *Elements* of Euclid (ca. 300 BCE) has remained, ever since, in one form or another, both a model for the presentation of mathematics and an important part of mathematical education. In modern research mathematics, too, subjects known as *differential geometry* and *algebraic geometry* are active and dynamic fields of study. The content of those modern subjects can seem very different because the connections with the past are not always apparent. When viewed through a historical lens, though, we see a fascinating story of how questions raised by close study of Euclid motivated new tools and new approaches that led directly to these later incarnations of geometry.

In this course, we will follow some of that historical development to identify and trace those fascinating connections through time.

- We will begin "at the beginning," with Euclid's geometry (especially Book I of the *Elements*), which will be familiar to most of you through your high school mathematical course work. We will focus there on the role of Euclid's *Postulate V* in the theory of *parallels*.
- We will note that it is possible to "do geometry" on the surface of the (two-dimensional) sphere and that the resulting spherical geometry is actually quite different from plane geometry. (This makes it somewhat hard to fathom part of the subsequent history because Greek mathematicians after the time of Euclid were very familiar with properties of spherical triangles and other aspects of that geometry!)
- We will see that after many attempts to "vindicate" Euclid by clarifying the status of Postulate V, in the early 19th century CE, through work of Janos Bolyai, Nikolai Lobachevsky (and Carl Friedrich Gauss), it became clear that there were alternate, *non-Euclidean* geometries– perfectly consistent mathematical theories on an equal footing with Euclidean geometry, but in which Postulate V does not hold.
- Starting with the work of René Descartes in the 16th century CE, and the subsequent development of calculus by Isaac Newton and Gottfried Leibniz, the use of real numbers to measure lengths and describe locations of points through coordinate geometry provided a collection of different ways to define geometric objects and study their properties.
- Work by Gauss on the intrinsic geometry of surfaces and Bernhard Riemann's *Habilitationsvortrag* led to a very general way of understanding geometry that showed how non-Euclidean geometries could be realized and opened the way to applications such as Albert Einstein's general theory of relativity.

Course Objectives

The major objectives of the course will be:

1) To examine the history and development of geometry through time, using some of the original sources to understand how and why those ideas were developed.
2) To make connections between different parts of your mathematical studies and serve as a "capstone" to your major program
3) To give you the experience of working on a substantial mathematical *project* leading to a written paper and oral presentation.

Texts

The main text book for the course is:

> J. McCleary, *Geometry from a Differentiable Viewpoint*, 2nd edition, Cambridge University Press.

A few other shorter readings will be distributed as .pdf files via the course homepage.

Course Schedule

A detailed day-by-day course schedule with due dates for assignments will be maintained on the course homepage. That listing is a tentative, evolving schedule, so you will probably want to refer to it frequently. Any important changes will also be announced in class well in advance.

Assignments and Grading

1) Midterm Exam (20 % of course grade) – tentative date: Friday, March 18.
2) 2 shorter quizzes (10 % of course grade) – tentative dates: Friday, February 19 and Friday, April 15
3) Final Project – paper (25 % of course grade), due: Monday, May 9, and oral presentation (15% of course grade) given in class, weeks of April 25 and May 2 (schedule to be determined later)
4) Problem Sets (about 8 through the semester – 20 % of course grade)
5) In-class group work (about 5 assignments – 10 % of course grade)

I will be keeping your course average in numerical form throughout the semester, and only converting to a letter for the final course grade. The course grade will be assigned according to the following conversion table (also see Note below):

- A – 94 and above
- A- – 90 - 93
- B+ – 87 - 89
- B – 84 - 86
- B- – 80 - 83
- C+ – 77 - 79
- C – 74 - 76
- C- – 70 - 73
- D+ – 67 – 69
- D – 60 - 66
- F – 59 and below.

Note: Depending on how the class as a whole is doing, some downward adjustments of the above letter grade boundaries may be made. No upward adjustments will be made, however. (This means, for instance, that an 85 course average would never convert to a letter grade of B- or below, although it might be a B+ in some circumstances.) If you ever have a question about the grading policy or your standing in the course, don't hesitate to ask me.

Advice On How To Succeed In This Class

A good "work ethic" is key. As you should be able to tell from the course description above, you do not need to be a "math genius" to do well in this course. But you will need to put in a consistent effort and keep up with the reading and assignments.

Come to class. Unless you are deathly ill, have a genuine family emergency, are away at a game or meet of a college athletic team, etc. plan on showing up here at 2:00 pm every Monday, Wednesday, and Friday this semester. Many of the class meetings will be structured around discussions or student presentations. Your participation is expected and needed for the success of the course!

Take notes and use them. This may seem obvious, but it is worth saying! Used intelligently, your notes can be a valuable resource as you work on problem sets and prepare for exams.

Use the text and class notes actively. Reading about mathematics is not like reading a novel. You will probably need to read and think over things more than once. You may want to work through examples or draw your own diagrams to understand some of the Euclidean proofs that we do.

Set up a regular study schedule and work at a steady pace. It's not easy to play catch-up in a mathematics course (even when the course is part of a first-year program with additional goals beyond the mathematics). You should expect to budget at least 6 hours in a typical week for work outside of class. The best way to use your time is to do a few problems, some reading from the books, and reviewing of class notes every day.

Most importantly, if you are having difficulty learning something, get help as soon as possible. You can do this by asking questions during class (any time something isn't clear), or seeing me during office hours.

Statement on Academic Integrity

All education is a cooperative enterprise between teachers and students. This cooperation works well only when there is trust and mutual respect between everyone involved. To be become an engaged and advanced learner, you must be able to think and work both independently and in concert with your peers. The College academic honesty policy states: "As an institution devoted to teaching, learning, and intellectual inquiry, Holy Cross expects all members of the College community to abide by the highest standards of academic integrity. Any violation of academic honesty undermines the student-teacher relationship, thereby wounding the whole community. The principal violations of academic honesty are plagiarism, cheating, and collusion.

Plagiarism is the act of taking the words, ideas, data, illustrative material, or statements of someone else, without full and proper acknowledgment, and presenting them as one's own.

Cheating is the use of improper means or subterfuge to gain credit or advantage. Forms of cheating include the use, attempted use, or improper possession of unauthorized aids in any examination or other academic exercise submitted for evaluation; the fabrication or falsification of data; misrepresentation of academic or extracurricular credentials; and deceitful performance on placement examinations. It is also cheating to submit the same work for credit in more than one course, except as authorized in advance by the course instructors.

Collusion is assisting or attempting to assist another student in an act of academic dishonesty."

The full statement on Academic Honesty in the College Catalog is available at

`http://www.holycross.edu/catalog/academic-honesty-policy.pdf`

While the temptation to engage in an act of academic dishonesty may arise because of time pressure, other events in your life, and so forth, using dishonest means to enhance a single grade is not worth the loss of your personal integrity. If in doubt about what you plan to do or write violates academic honesty, PLEASE ASK! If you do not know how to correctly cite reference materials, consult with me, the Writers Workshop, and/or visit the link below.
Department of History:

`http://www.holycross.edu/departments/history/website/academichonesty.htm`

Specific Guidelines for this Course

In this course, the examinations will be closed-book. No sharing of information with other students in any form will be permitted during exams. On group discussion write-ups, close collaboration with the other members of your group is expected. On the individual problem sets, discussion of the questions with other students in the class and with me during office hours is allowed, *even encouraged*. However, your final problem solutions should be prepared individually and the wording and organization of your final problem solutions should be entirely your own work. Moreover, if you do take advantage of any of the above options for discussion of problems with others, you will be required to state that fact in a footnote accompanying the problem solution. Failure to follow this rule will be treated as a violation of the College's Academic Integrity policy. For the papers, if you do consult a source other than the course texts, include a full reference in a bibliography section at the end of your paper, and identify any direct quotations. Information about the acceptable formats for doing this will be distributed with the paper assignments.