MATH 392 – Geometry Through History
Information on Final Projects
March 9, 2016

General Information

As announced in the course syllabus, one of the assignments for the seminar this semester will be a final project. You will be working on this project either individually, or in a *team of 2* (as you prefer). The goals will be to prepare a roughly 12-15 page research paper and an oral presentation to the class on your project. The presentations will be given the final class meetings of the semester – most likely May 2, 4, 6, and 9.

Schedule and Deadlines

- By Wednesday, March 23 – inform me which general topic you want to work on and whether you prefer to work alone, or who you will be working with. If your proposed topic has several different directions that might be pursued (see the descriptions below), please give an indication of which aspect you would like to work on. If you need assistance in forming "teams," I will be happy to help with that. Ideally, each group will work on a different project. In a few cases, the topics are large enough that if more than one group wants to try one, there will be ways to "split up" the topic into several parts. See me to discuss the possibilities.

- By Monday, April 18 – Submit, *by email*, a bibliography of sources to be used for your project. You should identify *at least six articles, books, or web sites*, that will be relevant. For each of your sources, write up a short paragraph giving a rough description of how that source relates to your main topic, what kind of information you will take from it, and how you will be using it (including a preliminary estimate of how reliable you think the information there is). The project descriptions below contain some first places to look, but you should plan to spend some time searching for additional sources of information. *Your final project papers may also refer to additional sources if you find that is necessary.*

- During the week of April 18, each team will meet with me during office hours (or at another time if that is not convenient) for a progress report and a chance to look at any questions that have come up as you have started to work on the project. I will be happy to discuss any aspect of the project at other times too, of course.

- We will decide which groups present which day when we get closer to the dates.

- All final project papers will be due no later than 5:00pm on *Monday, May 9*. (This is an absolutely firm deadline.)

Other Information

- Each of these projects will require a fairly substantial effort to identify sources of information. If you need to get books via interlibrary loan or get access to online journals, this may take some time. *Get started early!* Ms. Barbara Merolli, our Science Librarian, will be more than happy to assist you in the important process of assembling the resources you will use for your project.

- For the paper, you may use either Word (or an equivalent program), or the mathematical typesetting system called LaTeX. The presentations can be done with PowerPoint or LaTeX/Beamer (if you have used that before). I will be happy to help out with technical details and give you an introductory tutorial on LaTeX if needed.

- I will ask each group to do a "dry run" of their presentation with me at least one day before you go in front of the class. The purpose of this is to give you some feedback about what is working and what is not, and to give you some practice to minimize the effect of "nerves" when the time comes for the real thing.

The Final Project Report

This writing assignment will be comparable to assignments you have probably had in other social sciences or humanities courses and you should aim for the same standards in writing and in presenting your ideas that you did there. *In particular,*

- Distill the results of your investigations into a central argument. Show how different parts of the information you have learned support the points in that argument. A good research paper of this kind should be more than a series of "book reports" and more than just a compilation of all the different things you have read about or learned.

- The paper should be *well organized* and the writing should give the reader a clear indication where you are heading with your central argument at all times.

- Include a *References* section at the end including all books, articles, websites you used in the preparation of the work. For the websites, give the full URL, and the date you consulted. Any standard format is OK; I don't care so much about the format you use for the bibliography entries – the important thing is to document your sources and to give credit where it is due.

- Use footnotes or endnotes to identify the sources of the facts you present and be sure to identify any direct quotations or paraphrases that you use by a reference to one of the items in your References section. Any standard format is OK; I don't care so much about the footnoting format you use – again, the important thing is to document your sources and to give credit where it is due.

- Be clear, concise, and correct in your writing. Aim for *no typos, misspellings, or grammatical problems.* But even more importantly, each paragraph should have a clearly evident purpose in relation to your main argument.

- Equations, graphs, tables are OK but they should be used sparingly in the main text. (If you want to include more of these, that can be done in an additional Appendix section at the end.)

- Proofread your work carefully and have an "impartial" reader or readers look at it and give you comments. This can be one of the other teams or me. Be prepared and willing to *revise* your work based on the comments you get. Of course, this means that the writing must not be put off until the evening of May 8(!) Be sure you get started early enough so that the input can be put to productive use.

Ideas for Project Topics

More Historical Topics

The following topics would be primarily historical in nature. For these, you would need to learn about and present some mathematics that will be new to you. But the major focus of the paper and presentation would be more on how these individuals and their work affected the development of geometry through history. *One caution: Don't underestimate the challenge involved in some of these topics. Get started early.* It will quickly become very clear if you look back at English translations of original sources (or even modern discussions or reconstructions of older work) that understanding mathematics of the past is *often very difficult*. We are frequently coming at questions from very different starting points than the authors were, notation may be unfamiliar, and most importantly, the authors are not around any more to answer the inevitable questions that come up in trying to understand what they did(!)

(1) *Greek geometry after Euclid – Archimedes.* Archimedes (287 - 212 BCE) has consistently been regarded as probably the greatest mathematician and physicist in the ancient world. His geometric work used techniques such as Eudoxus's *method of exhaustion* to derive results on *mensuration* – computations of areas of plane figures, volumes of solids, and the like. Along the way, he developed quite accurate approximations to the constant π. One excellent focus for a project here would be the work known as *Quadrature of the Parabola*. In this book, Archimedes gives two different proofs of a formula for the area bounded by a chord of a parabola and the parabola itself. Both of them are fascinating and each anticipates later work involving (a) geometric series, and (b) integral calculus techniques for computing centroids of thin plates. In typical Greek fashion, the formula expresses this area as $\frac{4}{3}$ times the area of the triangle with one side equal to the chord, and third vertex the point on the parabola whose tangent line is parallel to the chord. This result is also discussed in another work of Archimedes called the *Method of Mechanical Theorems* which has had a really interesting history(!) It would be fun to look into that aspect too.

(2) *Greek geometry after Euclid – Apollonius.* Apart from Archimedes, probably the second greatest geometer from the generations after Euclid was Apollonius of Perga (262 - 190 BCE). Apollonius took the existing theory of conic sections (which also formed the subject of one of Euclid's works other than the *Elements*) and transformed and extended it immensely. In fact, we owe our modern names for the three types of conic sections–*ellipse, parabola, hyperbola*–to Apollonius's work called *Conics*. For this project, you would learn about how Apollonius defined these curves, how he studied them, what he proved about them, and how his work influenced later mathematicians.

A particularly interesting question is to what extent aspects of what Apollonius did can be said to anticipate the analytic (coordinate) geometry later developed by Descartes.

(3) *Geometry in the Medieval Islamic World.* For this topic, you would investigate the way the Greek mathematics of Euclid, Archimedes, Apollonius, and others was preserved in the Islamic world after the fall of the Western Roman Empire and the decline of the Eastern Roman (Byzantine) Empire. The initial phase of this included gathering existing manuscripts and translating them into Arabic. Much of this work occurred in Baghdad at a fascinating but somewhat enigmatic institution called the House of Wisdom founded by Caliph Harun al-Rashid (ca. 763 - 809 CE) and continued by his son the Caliph Abul Abbas Al Mamun (786 - 833 CE). We (and McCleary) mentioned several figures such as Umar Khayyam (1048 - 1131 CE), Nasir Al-Din Al-Tusi (1201 - 1274 CE) in the later part of this period when efforts were well under way to clarify the status of Euclid's Postulate V and extend what the Greeks had done. Some earlier European historians of mathematics claimed that what these mathematicians did was really only "preservation" of what the Greeks had achieved; they added nothing new of consequence. Is that point of view justified?

(4) *Descartes and La Géometrie.* In class, we talked briefly about the problem from the work of the Greek mathmetician Pappus of Alexandria that René Descartes (1596 - 1650 CE) used as a first example of the power of his method of coordinate (analytic) geometry. This appeared in a short volume called *La Géometrie* that Descartes wrote as a supplement to his philosophical *Discourse on the Method.* A good project topic would be to look at Descartes' geometrical work in more detail, discuss the other aspects of the way he essentially married algebra and geometry and launched the study of classes of curves defined by equations of particular forms. Descartes' influence can be traced, for instance, in the famous article of Isaac Newton on plane cubic curves. What were Newton's major results and how was he building on what Descartes had started?

(5) *Christopher Clavius, S.J. and the place of Euclid in early Jesuit education.* Among the early members of the Society of Jesus were several very impressive mathematicians, including especially Christopher Clavius, S.J. (1538 - 1612 CE). Among other achievements, Clavius prepared a version of a portion of Euclid's *Elements* that is somewhere between a translation and a reworking and reorganization with additional material added. This was used as a textbook quite widely for many years after his death. Clavius was also a vocal advocate for the inclusion of mathematics in the curriculum of the schools that the Jesuits were setting up all through Europe and in European colonies throughout the 17th century. An interesting topic would be to learn about the way that Clavius presented Euclid, the history of mathematics in early Jesuit education. Try to understand what education in mathematics in an early Jesuit college would have been like.

More Mathematical Topics

I have separated these topics (somewhat arbitrarily) from the first batch because I think the proper focus for these would be more on the mathematics itself and less on the historical development. You should still say something about the history, but the goal for these would often be to dig deeper into topics we started, or did background for, in class and take the subject farther.

(6) *Girolamo Saccheri, S.J. and his* **Euclid Freed of Every Flaw**. In class we discussed some of the beginnings of Saccheri's work (the Saccheri-Legendre theorem on the angle sum in a triangle, Saccheri quadrilaterals, their properties, etc.) In attempting to show that the HAA plus Postulates I-IV would yield a contradiction, Saccheri actually derived many true theorems in hyperbolic geometry. There is a starting discussion of this in Chapter 3 of McCleary (pages 37 - 41). But you should also track down the translation *Girolamo Saccheri's Euclides Vindicatus* by G.B. Halsted (yes, it's in English!) and study some of the other steps along the way to Saccheri's ultimate conclusion that what he had found was "repugnant to the nature of a straight line."

(7) *Cycloids, evolutes, involutes.* A beautiful chapter of the theory of plane curves involves what are known as cycloids, evolutes and involutes. You have probably seen the cycloid at some point defined as the path followed by one point on the rim of a wheel that rolls without slipping along a level path at a constant speed. Interestingly, this same cycloid curve occurs as the solution of two famous problems that interested mathematicians early in the history of calculus – the *brachistochrone* and *tautochrone* problems. Christiaan Huygens (1629 - 1695 CE) also saw that cycloids would be useful in the design of pendulum clocks. The mathematics he developed in studying this connection also led to the ideas of evolutes and involutes. The evolute of a plane curve $\alpha(s) = (x(s), y(s))$ can be defined the locus of centers of curvature of α. The cycloid has the special property that its evolute is another cycloid congruent to the original one(!) McCleary has a nice discussion of some of this starting on page 89. But you'll also want to look at other presentations to see some of the connections and how all these topics influenced the development of the theory of plane curves.

(8) *Map projections and geometry of surfaces.* Since ancient times, *cartography*, the art and science of constructing maps of regions of the Earth's surface, has been of interest. Maps help us plan land trips and sea voyages, and help us to help visualize the locations and relative distances between landmarks. The Greek-Roman mathematician Claudius Ptolemy (ca. 100 - 170 CE)), for instance, studied the problem of constructing maps in detail in his book *Geography.* This includes one of the largest databases of information surviving from the ancient world as the "raw material" for construction of a map of the world as known to Romans in his time. Classicists, including Prof. Neel Smith of the HC Classics Department, have used this to produce modern-style maps to see how well Ptolemy's data conforms to reality. In modern mathematical terms, McCleary says (Chapter 7^{bis}, page 138) that "the fundamental problem of cartography is to present a portion (a region) R of the globe, idealized as the unit sphere S^2, on a flat surface, that is, to give a mapping $Y : (R \subset S^2) \to \mathbb{R}^2$ that is one-one and differentiable, and that has a differentiable inverse on $Y(R)$." The mapping Y is often called a *map projection.* There are several possible projects here.

- One would be to learn about the two different map projections that Claudius

Ptolemy constructed and perhaps learn how his data can be used to generate an actual map. This could get really interesting if you are ambitious(!)

- Another project would be to understand some of the common map projection functions that are used to produce actual maps we use – stereographic, central (gnomonic), and cylindrical (e.g. Mercator) projections. As you have probably noticed, common map projections like the Mercator projection produce *distortions*. For example, regions near the poles are shown with much greater area than they should have. But in fact, it is a theorem that *no perfect map projection exists* – every such Y distorts *something*: angles between curves on the globe, lengths of curves on the globe, areas of regions, or all three.

(9) *The Poincaré disc and upper half-plane models.* (This topic is recommended mainly for students who may be taking Complex Analysis this term). One of the great anecdotes about mathematical inspiration comes from the French mathematician Henri Poincaré (1854 - 1912 CE). At one point, Poincaré was deeply involved in trying to solve a problem in complex analysis regarding what he called "Fuchsian functions" (functions defined on the unit disc in the complex plane that would satisfy particular types of transformation rules). He says:

Just at this time I left Caen, where I was then living, to go on a geological excursion under the auspices of the school of mines. The changes of travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non- Euclidean geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake I verified the result at my leisure.

As a result he saw how to create *analytical models* of hyperbolic geometry using the interior of the unit disc and the upper half-plane in the complex plane, and do all of the hyperbolic constructions we did in terms of complex analysis(!) For this project, you would learn about these models of hyperbolic geometry and explain how they show that hyperbolic geometry is consistent if and only if Euclidean geometry is consistent(!)

(10) *A topic of your own choice.* If there is another topic you would prefer to work on, I am open to suggestions. This could be something suggested by a topic we have seen in this course, something you have seen in another course, or a section of McCleary's book that we will not have time to cover. If you want to propose a topic of your own, *you must get my approval* before starting to work. For the March 23 deadline above, write up a short description of the topic or questions you want to look at and how you want to try to address them. I will let you know as soon as possible whether you have my approval.