

with(Groebner);

[Basis, FGLM, HilbertDimension, HilbertPolynomial, HilbertSeries, Homogenize, InitialForm, InterReduce, IsBasis, IsProper, IsZeroDimensional, LeadingCoefficient, LeadingMonomial, LeadingTerm, MatrixOrder, MaximalIndependentSet, MonomialOrder, MultiplicationMatrix, MultivariateCyclicVector, NormalForm, NormalSet, RationalUnivariateRepresentation, Reduce, RememberBasis, SPolynomial, Solve, SuggestVariableOrder, Support, TestOrder, ToricIdealBasis, TrailingTerm, UnivariatePolynomial, Walk, WeightedDegree] **(1)**

$Id := [c^2 + s^2 - 2^2, (u - 4)^2 + v^2 - 3^2, (c - u)^2 + (s - v)^2 - 4^2, (x - c)^2 + (y - s)^2 - 3^2, (x - u)^2 + (y - v)^2 - 3^2];$

$Id := [c^2 + s^2 - 4, (u - 4)^2 + v^2 - 9, (c - u)^2 + (s - v)^2 - 16, (x - c)^2 + (y - s)^2 - 9, (x - u)^2 + (y - v)^2 - 9]$ **(2)**

$New := [(x - c) \cdot (v - s) - (y - s) \cdot (u - c) - 4 \cdot \sqrt{5}];$

$New := [(x - c) (v - s) - (y - s) (u - c) - 4 \sqrt{5}]$ **(3)**

$IdNew := [op(Id), op(New)];$

$IdNew := [c^2 + s^2 - 4, (u - 4)^2 + v^2 - 9, (c - u)^2 + (s - v)^2 - 16, (x - c)^2 + (y - s)^2 - 9, (x - u)^2 + (y - v)^2 - 9, (x - c) (v - s) - (y - s) (u - c) - 4 \sqrt{5}]$ **(4)**

$Basis(IdNew, lexdeg([c, s, u, v], [x, y]));$

$[3600 + 832 x^2 y^2 - 216 x y^2 - 216 x^3 - 1631 x^2 + 3745 y^2 + 32 \sqrt{5} y^5 + 544 \sqrt{5} y^3 + 2560 \sqrt{5} y + 32 \sqrt{5} x^4 y + 64 \sqrt{5} x^2 y^3 - 256 \sqrt{5} x^3 y - 256 \sqrt{5} x y^3 + 544 \sqrt{5} x^2 y - 768 \sqrt{5} x y + 16 y^6 + 48 x^4 y^2 + 48 x^2 y^4 - 192 x^5 + 672 x^4 - 384 x^3 y^2 - 192 x y^4 + 160 y^4 - 1160 x + 16 x^6, 928 x y^3 + 2592 v y^2 - 3248 y^3 + 32 \sqrt{5} x^5 - 400 \sqrt{5} x^4 - 288 \sqrt{5} y^4 + 1472 \sqrt{5} x^3 - 232 \sqrt{5} x^2 - 3352 \sqrt{5} y^2 - 4946 \sqrt{5} x + 64 \sqrt{5} x^3 y^2 + 32 \sqrt{5} x y^4 - 688 \sqrt{5} x^2 y^2 + 1472 \sqrt{5} x y^2 + 3312 \sqrt{5} v y - 80 x^4 y + 928 x^3 y - 3248 x^2 y - 1800 v x + 1164 x y - 80 y^5 - 160 x^2 y^3 + 7200 v - 1080 \sqrt{5} - 11125 y, -32 \sqrt{5} x^4 y - 64 \sqrt{5} x^2 y^3 - 32 \sqrt{5} y^5 + 112 \sqrt{5} x^3 y + 112 \sqrt{5} x y^3 - 80 x^5 - 160 x^3 y^2 - 80 x y^4 + 256 \sqrt{5} x^2 y + 256 \sqrt{5} y^3 + 928 x^4 + 928 x^2 y^2 + 720 \sqrt{5} v x - 3120 \sqrt{5} x y + 2592 v x y - 3104 x^3 - 3104 x y^2 - 2880 \sqrt{5} v + 1598 \sqrt{5} y - 3384 y v + 40 x^2 - 1124 y^2 + 9827 x + 3060, -4640 x y^3 + 12960 v x^2 + 9760 y^3 - 160 \sqrt{5} x^5 + 2000 \sqrt{5} x^4 + 1440 \sqrt{5} y^4 - 9952 \sqrt{5} x^3 + 10232 \sqrt{5} x^2 + 25832 \sqrt{5} y^2 + 15334 \sqrt{5} x - 320 \sqrt{5} x^3 y^2 - 160 \sqrt{5} x y^4 + 3440 \sqrt{5} x^2 y^2 - 9952 \sqrt{5} x y^2 - 13968 \sqrt{5} v y + 400 x^4 y - 4640 x^3 y]$ **(5)**

$$\begin{aligned}
& + 9760 x^2 y - 42840 vx - 5820 xy + 400 y^5 + 800 x^2 y^3 - 36000 v + 63720 \sqrt{5} \\
& + 159305 y, 144 \sqrt{5} x^2 y + 144 \sqrt{5} y^3 + 80 x^4 + 160 x^2 y^2 + 80 y^4 - 576 \sqrt{5} vx \\
& - 640 x^3 - 640 xy^2 - 800 \sqrt{5} v + 1992 \sqrt{5} y - 1440 yv + 1376 x^2 + 1376 y^2 \\
& + 776 u + 2820 x + 2413, 80 \sqrt{5} x^4 + 160 \sqrt{5} x^2 y^2 + 80 \sqrt{5} y^4 - 640 \sqrt{5} x^3 \\
& - 640 \sqrt{5} xy^2 - 1440 \sqrt{5} vy + 1376 \sqrt{5} x^2 + 1376 \sqrt{5} y^2 + 720 x^2 y + 720 y^3 \\
& + 3596 \sqrt{5} x - 2880 vx + 2413 \sqrt{5} + 1746 s - 4194 v + 8408 y, 144 \sqrt{5} x^2 y \\
& + 144 \sqrt{5} y^3 + 80 x^4 + 160 x^2 y^2 + 80 y^4 - 576 \sqrt{5} vx - 640 x^3 - 640 xy^2 \\
& + 2304 \sqrt{5} v - 1112 \sqrt{5} y - 1440 yv + 1376 x^2 + 1376 y^2 + 6984 c - 3388 x \\
& + 2413, 32 \sqrt{5} x^2 y + 32 \sqrt{5} y^3 + 16 x^4 + 32 x^2 y^2 + 16 y^4 - 256 \sqrt{5} vx \\
& - 192 x^3 - 192 xy^2 - 320 \sqrt{5} v + 704 \sqrt{5} y + 576 v^2 - 1088 yv + 816 x^2 \\
& + 816 y^2 + 104 x - 1375]
\end{aligned}$$