

Mathematics 357 – Combinatorics  
Discussion 2 – Poker Hands  
February 13, 2017

*Background and Goals*

“Poker” is really a whole family of card games with a multitude of variations and “wrinkles.” Frequent poker players will, in fact, often invent new variations to make the play of the game more interesting and challenging. All versions of the game involve

- dealing a “hand” (= some number of cards from the standard deck) to each player,
- estimating the value of the hand by its composition (i.e. the denominations and suits of the cards it contains), then
- betting (or not) on how you think your hand might compare with the hands held by the other players. (If you play poker, you know that there are also many psychological aspects of the game at this point – you can try to “bluff” the other players into thinking you have a better hand than you do, etc.)

To keep things simple today, we will consider only the case of hands consisting of *5 cards*, as in many of the simpler versions of the game. We will also consider only the following ordering of the denominations of the cards

$$2 < 3 < 4 < \cdots < 10 < J < Q < K < A,$$

that is aces are always going to be the highest cards in their respective suits (they will *not* also serve as the “1” in the suit in some combinations of cards). Each suit consists of one card of each denomination and there are 4 suits – clubs, diamonds, hearts, and spades,  $4 \times 13 = 52$  cards in all.

*Questions*

- A) How many different 5-card hands are possible dealing from the standard deck? *Hint:* It’s just which five cards are in the hand that matters, not how they are arranged or ordered.
- B) The lowest hand that is worth anything in most poker games is a hand with exactly *one pair* of cards of the same denomination (say 8’s), then three other cards which are in three distinct denominations (so no second pair), *and* different from the denomination of the pair (so no three of a kind). How many different one-pair hands are possible?
- C) The most valuable hands in many versions of poker are called *royal flushes* – these are five-card hands consisting of the 10, *J, Q, K, A* cards all in the same suit. How many different royal flushes are there?
- D) A *straight flush* is a hand that has five cards in the same suit and in consecutive denominations, but is not a royal flush. How many different straight flushes are there?

- E) A *flush* is a hand that has five cards in the same suit, but is not a straight flush or a royal flush. How many such hands are there?
- F) A *straight* is a hand that has five cards in consecutive denominations, but is not a straight flush or a royal flush. How many such hands are there?
- G) Based on your answers to parts E and F, should a flush beat a straight or vice versa? Explain. Hint: Rarer hands should be more valuable.

*Assignment*

Group writeups due Wednesday, February 15.