College of the Holy Cross
MATH 133, section 1 – Calculus with Fundamentals
Final Exam – December 7, 2015

General Information

- The final examination will be given 11:30am to 2:00pm on Tuesday, December 15, in Swords 321 (where our class has met on Mondays and Fridays).
- The final will be similar in format to the midterm exams but perhaps 1.5 times as long. If you are well prepared and you work steadily, then you should be able to finish the exam in about 1.75 hours. However, you will have the full 2.5 hour period to work on the exam if you need that much time.
- The final will be a *comprehensive exam* – it will cover all the topics we have studied this semester, with roughly equal weight given to the four sections of the course corresponding to the four midterm exams.
- The review sheets for the four midterms are reposted on the course homepage if you need another copy of any of them.
- It may not be necessary to say this, but here goes anyway: *You should take this exam seriously* – it is worth 20% of your course average and it can pull your course grade up *or down* depending on how you do.
- Get started reviewing over the weekend. Don't try to "cram" the night before.
- Review the videos, your class notes, and the text, especially for topics where you lost points on the midterms. There are a lot of worked-out examples and discussions of all of the topics we have covered there.
- Be sure you actually do enough practice problems so that you have the facility to solve exam-type questions in a limited amount of time. *Even if you have saved solutions for practice problems from the midterms*, it is going to be much more beneficial to do practice problems starting "from scratch" rather than just reading old solutions.
- You may use a calculator, but no graphing features. No cell-phones, computers, or other electronic devices beyond a basic calculator may be used during the exam. Please do not bring them with you; they will be subject to confiscation for the period of the exam if you use them.
- Lauren is arranging a review session before the exam. We can discuss this in class on Friday, December 11.
- I will also be available 8 - 10am and 3 - 5pm on Monday, December 14 for "last minute" questions.
- The following is (a slightly edited version of) the final I gave the last time I taught a Calculus 1 course, in fall 2013. It's a good guide for what our exam will look like.

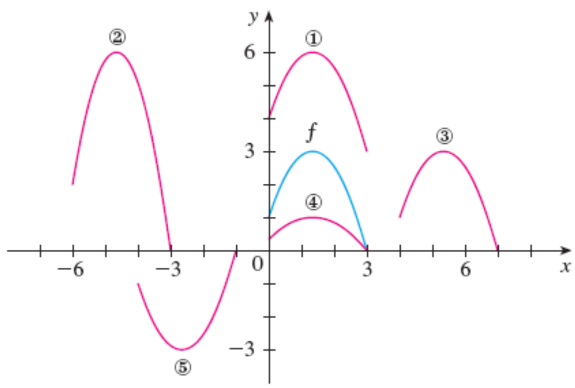

Figure 1: Figure for problem I

I. The graph $y = f(x)$ is given in blue (more like cyan). Match each equation with one of the numbered pink (actually, magenta) graphs.

A) $y = f(x-4)$ is plot number: ________

B) $y = f(x) + 3$ is plot number: ________

C) $y = \frac{1}{3}f(x)$ is plot number: ________

D) $y = -f(x+4)$ is plot number: ________

E) $y = 2f(x+6)$ is plot number: ________

II. A cup of hot chocolate is set out on a counter at $t = 0$. The temperature of the chocolate t minutes later is $C(t) = 70 + 80e^{-t/3}$ (in degrees F).

A) What is the temperature of the chocolate at $t = 0$?

B) What is the rate of change of the temperature at $t = 10$ minutes?

C) How long does it take for the temperature to reach $100°F$?

III. Compute the following limits. Any legal method is OK.

(A) $\displaystyle\lim_{x\to 3} \frac{x^2 + x - 12}{x^2 - 5x + 6}$.

(B) $\displaystyle\lim_{x\to 1^-} \frac{|x-1|}{x^2 - 1}$.

(C) $\displaystyle\lim_{x\to 0} \frac{\tan(x)}{x}$

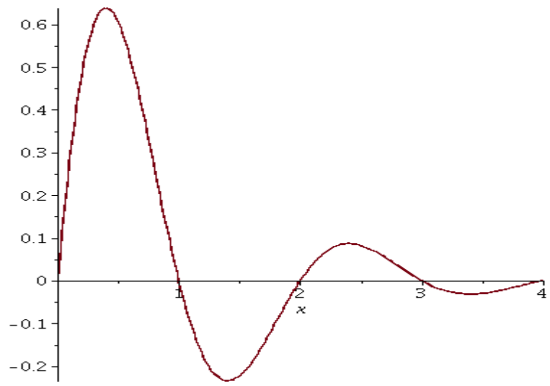

Figure 2: Figure for problem V.

IV.

A) *Using the limit definition*, and showing all necessary steps to justify your answer, compute $f'(x)$ for $f(x) = 5x^2 - x + 3$.

Using appropriate derivative rules, compute the derivatives of the following functions. You do not need to simplify your answers.

B) $g(x) = 4x^3 + \sqrt{x} + \dfrac{2}{\sqrt[4]{x}} + e^2$

C) $h(x) = \dfrac{\sin(x) + x}{\sec(x)}$

D) $i(x) = \ln(x^3 + 3)$

E) $j(x) = \tan^{-1}(12x + 2)$

V. The graph in Figure 2 shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $0 \le x \le 4$. Note: This *is not* $y = f(x)$, it is $y = f'(x)$. Using the graph, *estimate*

A) The interval(s) on which $f(x)$ is increasing.

B) The critical points of $f(x)$ *in the open interval* $(0, 4)$. Say what the behavior of $f(x)$ is at each critical number (local max, local min, neither).

C) The interval(s) on which $y = f(x)$ is concave down.

VI. A town wants to build a pipeline from a water station on a small island 2 miles from the shore of its water reservoir to the town. One possible route is shown dotted in red. The town is 6 miles along the shore from the point nearest the island. It costs \$3 million per mile to lay pipe under the water and \$2 million per mile to lay pipe along the shoreline.

A) Give the cost $C(x)$ of the pipeline as a function of the location x as shown.

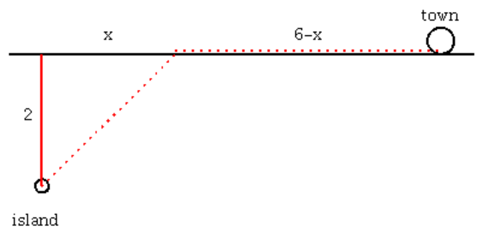

Figure 3: Figure for problem VI.

B) Where along the shoreline should the pipeline hit land to minimize the costs of construction? Say how you know this gives the minimum cost.

VII. A block of dry ice (solid CO_2) is evaporating and losing volume at the rate of 10 cm^3/min. It has the shape of a cube at all times. How fast are the edges of cube shrinking when the block has volume 216 cm^3?

VIII. True or false: The graph obtained by stretching $y = e^{-x}$ vertically by a factor of 2 can *also* be obtained from $y = e^{-x}$ by a horizontal shift. Explain your answer.