MATH 133 – Calculus with Fundamentals 1
Logarithm Functions
September 21, 2015

Background

As my high school math teacher Mr. Brennan said, *"a logarithm is an exponent."* He was a total nerd, but he was right! Of course, this is another way of saying the key property defining the $\log_b$ function:

$$y = \log_b x \Longleftrightarrow x = b^y,$$

so $\log_b(b^y) = y$ for all real y, and $b^{\log_b(x)} = x$ for all $x > 0$. The most important logarithm function for us is the *natural logarithm* $\ln(x)$, which is the same as $\log_e(x)$ where $e \doteq 2.71828\cdots$ is (for now) a somewhat mysterious number(!) From the properties of exponents, (see the table on page 41 of our text), we get the key properties of the ln function:

(1) $\ln(x_1 x_2) = \ln(x_1) + \ln(x_2)$,

(2) $\ln\left(\frac{x_1}{x_2}\right) = \ln(x_1) - \ln(x_2)$, and

(3) $\ln\left(A^B\right) = B\ln(A)$,

and similarly for $\ln_b$ for any $b > 0$. Today, we want to practice using these properties of the logarithm functions to simplify expressions and solve equations.

Questions

1) Simplify and compute the value. (Do these *without using a calculator*!)
   (a) $\log_2\left(\frac{1}{128}\right)$
   (b) $\log_5(125)$
   (c) $\ln(e^4) + \ln\left(\frac{1}{e^5}\right)$

2) Solve each of these equations for the unknown by using properties of logarithms. Get an exact answer (expressed using logarithms), then find a decimal approximation using a calculator.
   (a) $e^{5x+1} = 2$.
   (b) $2^{2x} = 3^{3x-5}$.
   (c) $e^{2t+1} = 7^{t-4}$

3) (a) Explain why $f(x) = e^{2(x-1)} + 1$ has an inverse function. (Hint: What is the property that says a function has an inverse function? Sketch the graph $y = e^{2(x-1)} + 1$ to see that this function "has it.")
   (b) Find a formula for f^{-1} for the function from part (a). Sketch the graph $y = f^{-1}(x)$ on the same axes as you had in part (a).