

MATH 133 – Calculus with Fundamentals 1
Piecewise-defined functions
September 14, 2015

Background

In the video for today, we saw a real-world example of the idea of a function defined “piecewise” by different formulas on different sections of its domain. In today’s session, you will work with several more examples of these.

Questions

- 1) For each of the following piecewise-defined functions $f(x)$, sketch the graph and determine the domain and range. (Note: If values of x are not covered in any of the stated “pieces” or cases, they are *not contained* in the domain. Each “piece” of these examples will be types of functions we have seen before, so you can plot them using ideas we have discussed before.)

- (a) The function

$$g(x) = \begin{cases} 1 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

- (b) The function

$$f(x) = \begin{cases} x^2 & \text{if } -3 < x \leq 0 \\ -x^2 & \text{if } 0 < x < 3. \end{cases}$$

- (c) The function

$$g(x) = \begin{cases} x^2 + 2x & \text{if } x < 2 \\ -4x + 9 & \text{if } x \geq 2. \end{cases}$$

- 2) Each of the following scenarios describes a car trip along a straight road joining two cities 120 miles apart. Mathematically, the road will be the real number line, with your home city at $x = 0$ and the other city is at $x = 120$. For each of the following scenarios, draw a graph of the position of your car as a function of time that describes it, then try to determine piecewise formula(s) for x as a function of t that describe each one. You’ll need to use the equation $\text{rate} \times \text{time} = \text{distance}$ for straight line motion at a constant speed.

- (a) You travel at constant speed of 45 miles per hour for one hour then, realizing you are going to be late, you travel the rest of the way at a constant speed of 75 miles per hour for one hour.
- (b) You travel at a constant speed of 60 miles per hour for one hour, stop for half an hour while a large flock of sheep cross the highway, then continue on at a constant speed of 60 miles per hour until you reached the other city.
- (c) You travel for one hour at a constant speed 60 miles per hour, then are engulfed in a dense fog. You turn around and drive home at a constant speed of 30 miles per hour.