

MATH 133 – Calculus with Fundamentals 1
Higher Derivatives
November 3, 2015

Background

If $f(x)$ is a function, $f'(x)$ is often called its *first derivative*. (In the alternate notation that might be written $\frac{dy}{dx}$ if we're thinking of the graph $y = f(x)$). The reason for this is that it is possible to go on and differentiate $f'(x)$ to get another new function. The derivative of $f'(x)$, that is, $(f')'(x)$ is also called the *second derivative* of the original f , and written $f''(x)$ or $\frac{d^2y}{dx^2}$. Continuing in the same way, if we can differentiate $f''(x)$, the result is called the *third derivative* of f , and so forth. The rules for computing these *higher derivatives* are exactly the same as the rules for computing $f'(x)$ to start. Today, we want to practice with these and understand why they are interesting.

Questions

- (1) For each function, use the appropriate short-cut rules to find the first derivative, and then differentiate again to get the second derivative:
 - (a) $f(x) = x^5 + 4x^3 + x$. Also find the third derivative $f'''(x)$, the fourth derivative, the fifth derivative, and the sixth derivative for this one. (What *always* happens if you differentiate a polynomial function repeatedly enough times?)
 - (b) $g(x) = \frac{x}{x^2 - 1}$. Your life will be a lot easier here if you simplify the first derivative before differentiating again to get $g''(x)$.
 - (c) $h(x) = (x^2 + x + 1)e^x$. Also find the third derivative $h'''(x)$ for this one.
- (2) So *why* would we want to be able to differentiate multiple times? The answer is that the second derivative f'' in particular encodes interesting information about the original function f .
 - (a) Suppose we know $f''(x) > 0$ on some interval (a, b) . Recall that $f'' = (f')'$. What can we say about f' on that interval? Draw pictures illustrating graphs on which $f''(x) > 0$ for all x . What is the name for the property you are seeing (recall today's video)?
 - (b) Now, suppose we know $f''(x) < 0$ on some interval (a, b) . Recall again that $f'' = (f')'$. What can we say about f' on that interval? Draw pictures illustrating graphs on which $f''(x) < 0$ for all x . What is the name for the property you are seeing?
- (3) Do Problem 39 on page 154 of our text.